Pre-lab 11: Operational Amplifiers, Part II

Today we continue our study of op-amps with some useful circuits. Our starting point is the "inverting" amplifier from the last lab, shown in Figure 1.

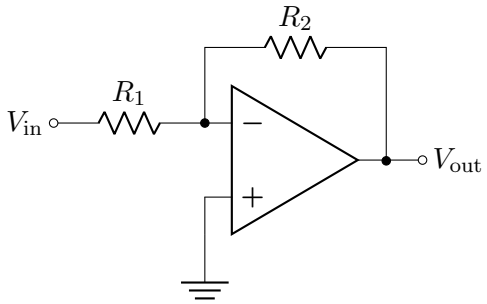


Figure 1: Inverting amplifier

Let's recap how this circuit worked:

1. The non-inverting input is grounded. Its voltage is 0 V.
2. Negative feedback (Golden Rule I) makes the inverting input voltage close to 0 V.
3. The input current is $I_{\text{in}} = \dfrac{V_{\text{in}}}{R_1}$.
4. Because of Golden Rule II, the same current I_{in} flows through both R_1 and R_2.
5. The output voltage is $V_{\text{out}} = -R_2 I_{\text{in}} = -\dfrac{R_2}{R_1} V_{\text{in}}$.
6. The closed-loop gain is $\dfrac{V_{\text{out}}}{V_{\text{in}}} = -\dfrac{R_2}{R_1}$.

The key is Step 2, where the op-amp is actively maintaining its inverting input at 0 V without drawing any current into it. We can use the same basic idea to build interesting circuits by replacing R_1 and R_2 with different elements.

Summing Amplifier

Let's duplicate the input of the inverting amplifier, as shown in Figure 2. The negative feedback is still

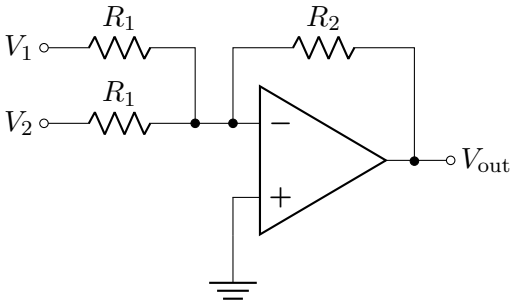


Figure 2: Summing amplifier

keeping the inverting input at 0 V. The currents from the two inputs are $I_1 = V_1/R_1$ and $I_2 = V_2/R_1$. The sum of I_1 and I_2 flows through R_2, which makes the output voltage

$$V_{\text{out}} = -(I_1 + I_2) \cdot R_2 = -\frac{R_2}{R_1} \cdot (V_1 + V_2). \tag{1}$$

The output voltage of this circuit — a *summing amplifier* — is proportional to the sum of the two input voltages.

(Perfect) Integrator

Back in Lab 3, we saw an approximation to a functional integrator, in the form of a passive RC (same configuration as the RC lowpass filter). It worked, sort of, as long as the output voltage was far smaller than the input voltage. We promised we'd learn later how to make a perfect integrator. Well, that "later" is *now*!

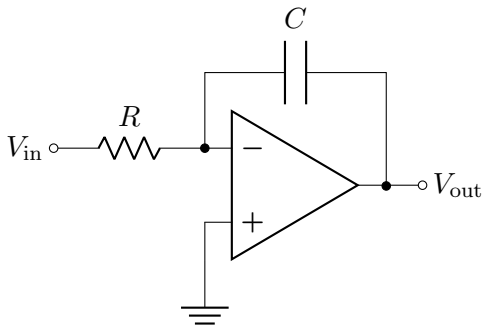


Figure 3: Perfect integrator circuit

We take the inverting amplifier and replace the feedback resistor with a capacitor, as shown in Figure 3. Even though we have a capacitor (instead of a resistor) between the op-amp's inverting input and the output, it still provide the negative feedback so that $V_{\text{in}-} \approx V_{\text{in}+} = 0$ is maintained. The input current through R is $I_{\text{in}} = V_{\text{in}}/R$, which flows into the capacitor and increases the charge. The charge Q is related to the voltage across the capacitor by $Q = C(V_{\text{in}-} - V_{\text{out}})$. Therefore

$$\frac{V_{\text{in}}}{R} = \frac{dQ}{dt} = -C\frac{dV_{\text{out}}}{dt}. \tag{2}$$

Integrating with time gives

$$V_{\text{out}} = -\frac{1}{RC}\int V_{\text{in}} dt. \tag{3}$$

It is an integrator.

What's nice about this circuit is that it's "perfect" — the output can be larger than the input, or of opposite sign, or whatever. We will see in the lab that the degree of perfection depends on several properties of the op-amp that we've swept under the rug for now. But, as a general statement, the op-amp integrator is really quite spectacularly good.

If you have been paying attention to the math above, you might be wondering what happened to the constant of integration. Shouldn't V_{out} be

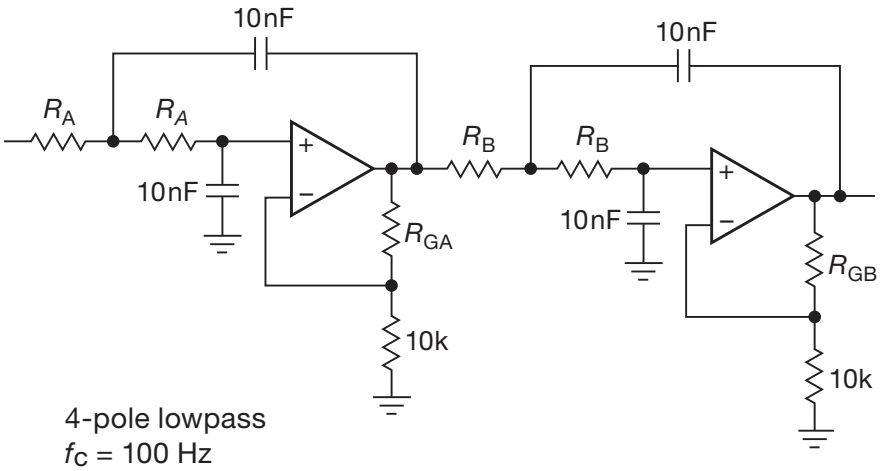
$$V_{\text{out}} = -\frac{1}{RC}\int V_{\text{in}} dt + \textbf{const?}$$

The answer is yes: the output voltage of a perfect integrator has an arbitrary DC offset. Another way of looking at it is that the feedback capacitor has an infinite impedance at frequency $f = 0$ Hz. That means the negative feedback does not work at DC, and the output voltage is undetermined. We will discuss how to remedy this problem in the lecture.

Active filter

Back when we learned about passive filters, we saw that the simple RC forms a lowpass or highpass filter, with gentle rolloff characteristics. Filters with steeper rolloffs can be designed by combining capacitors and inductors, but they are more difficult to implement in practice because inductors depart from the ideal (they have internal resistance and capacitance, and are troublesome to adjust).

Happily, nearly ideal filters can be made by combining capacitors (which adhere closely to ideal properties) with op-amps. These are known as *active filters*.



Filter Type	R_A	R_{GA}	R_B	R_{GB}	Gain
Bessel	110k	845Ω	100k	7.68k	1.91
Butterworth	158k	1.54k	158k	12.4k	2.57
Chebyshev (0.5dB)	267k	5.76k	154k	16.5k	4.21

Figure 4: A "VCVS (Voltage Controlled Voltage Source) 4-pole lowpass filter," with component values for several filter response types — see Fig. 5, where frequency responses are shown for filters with one more op-amp. (It's nicely typeset, because we shamelessly stole it from *the Art of Electronics*.)

You can easily spend a full semester learning about active filters. Here we dip one toe into the subject, displaying an example filter circuit and its frequency response (Fig. 4). If you want to know more, and want to build filters for a particular task, take a look at *the Art of Electronics*, Chapter 6.

Know, also, that you can get complete active filters as commercial integrated circuits: most of the hard work has been done for you, including on-chip integration of closely matched R's and C's. Such "filter ICs" come in two varieties: *continuous time* and *switched capacitor*.

Continuous time filters are analog circuits made from op-amps, resistors, and capacitors (like Fig. 4); the filter characteristics are set by the component values, and of course the circuit configuration. The thing just sits there and acts like a filter. *Switched-capacitor filters* use a capacitor combined with a MOSFET switch, turned on and off by an externally applied clocking signal, to substitute for the input

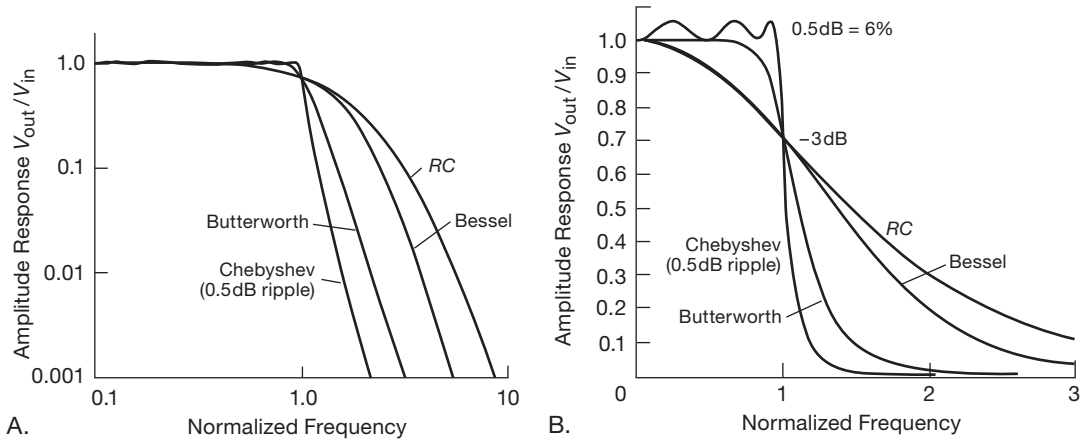


Figure 5: Comparison of some common 6-pole lowpass filters. The same filters are plotted on both linear and logarithmic scales.

resistor in the classic op-amp integrator. The effective resistor value is set by the clocking frequency. A typical switched-capacitor filter uses several such integrators, in combination with additional op-amps, to implement the desired filter function. Switched capacitor filters have the advantages of being simply tuned over a wide range (via the applied clocking frequency), of maintaining stable characteristics, and of being particularly easy to fabricate as integrated circuits. However, they are generally noisier (i.e., with smaller dynamic range), have higher distortion, and can introduce switching artifacts such as aliasing and clock feedthrough.

Pre-lab 11 Hand-in

Name: _____

Pre-lab 11.1 How can one modify the summing amplifier in Figure 2 so that the output voltage V_{out} is proportional to the *weighted* sum of two input voltages V_1 and V_2? Draw a diagram below and choose the component values so that

$$V_{\text{out}} = -\left(V_1 + \frac{V_2}{10}\right)$$

is achieved.

From the Last Lab:

Pre-lab 11.2 Post-lab 8.1 You have a sensor whose DC-coupled output voltage ranges over $\pm 20\,\text{mV}$, and whose source resistance is $100\,\text{k}\Omega$. Draw an op-amp circuit to amplify this signal so that it can drive a data logging instrument that wants a signal that goes to $\pm 5\,\text{V}$, and that presents an input impedance of $10\,\text{k}\Omega$. Ensure that your circuit does not load the sensor's output by more than 1%, and that it can drive the data logger without having your circuit's output signal attenuated by more than 1%. [Hint: treat these as two separate problems to solve when you're desiging your circuit. First deal with the gain problem, and then think about the sensor load problem.]

Draw your circuit in the dashed box in the figure below.

