Pre-lab 2: Input/Output Impedance

Thévenin Equivalent Circuit

There's More to Voltage than Just Voltage

The simplest collection of resistors, two resistors in series, constitutes a *voltage divider*, because the total voltage across the two resistors is "divided" between them, with the division proportional to the two resistances:

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{R_2}{R_1 + R_2}.$$

So, in Figure 1 we expect to see 5 V at the output of circuit A (and likewise for circuit B, which is identical, but drawn in the normal ground-referencing way).

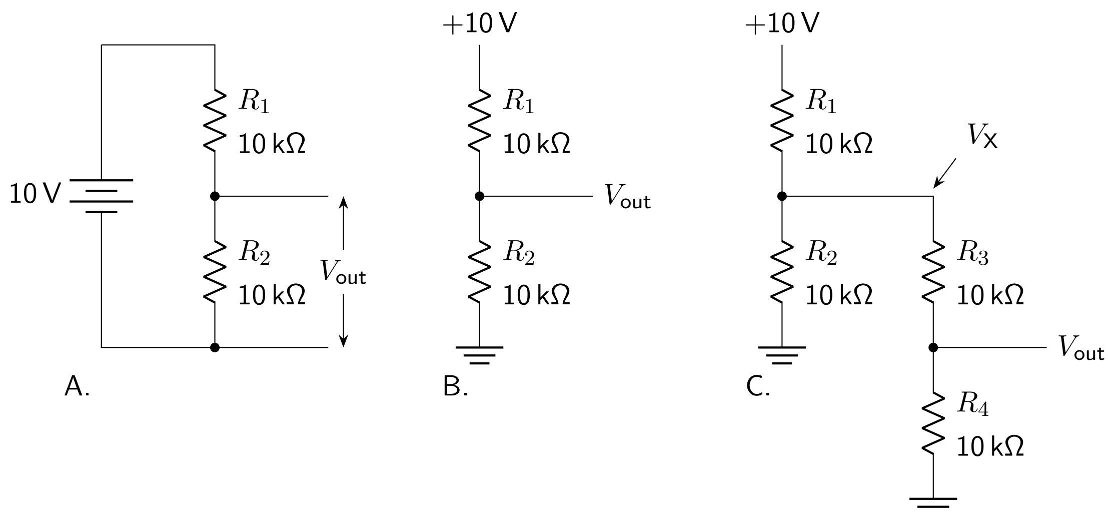


Figure 1: Three voltage dividers. A and B are the same circuit: A is drawn the Physics 15B way, whereas B implicitly references the common potential "ground." C is a cascade of two identical dividers, but with a surprise.

But what about circuit C? Both dividers are divide-by-two (equal top and bottom resistances), so naively you might think "point V_X is at 5 V, and so V_{out} is 2.5 V." But, you could instead work out circuit C by figuring the parallel resistance of R_2 combined with R_3 and R_4. Here's what you'd get:

$$R_2 \parallel (R_3 + R_4) = \frac{1}{\frac{1}{R_2} + \frac{1}{R_3+R_4}} = 6.66\,\text{k}\Omega.$$

Call that R_2', the new lower resistor of a new 2-resistor divider. Now what output voltage V_{out} do we get? Well, the potential V_X at the connection between R_1 and R_2' is

$$V_X = V_{\text{in}} \cdot \frac{R_2'}{R_1 + R_2'} = 4\,\text{V}, \quad \text{which leads to} \quad V_{\text{out}} = V_X \cdot \frac{R_4}{R_3 + R_4} = 2\,\text{V}.$$

Not what we had guessed.

One lesson from this simple example is that a voltage divider is not a very good "battery" — its output (in this case 5 V) droops (to 4 V) when "loaded" with another circuit (in this case an identical voltage divider, whose output is then 2 V).

Thévenin's Equivalent

But there's another important lesson: saying what the voltage is at some point in a circuit (a "node," in circuit language) does not tell you everything you need to know. In this example we can rely on the 10 V from the battery, but evidently we cannot rely on the 5 V output from circuit B, because when loaded with 20 kΩ its output voltage (V_X) dropped to 4 V.

Not surprising, you'd say: the stuff hooked onto the R_1R_2 divider changed the circuit, as we just calculated. And, that's correct. You can always consider a circuit modified when you tie something onto its output, and go ahead and re-do the circuit calculations.

But there's a powerful, and way better, way to deal with this. It turns out that any 2-terminal network consisting of resistors and batteries (it can have multiple batteries, and a tangle of interconnected resistors — a real Jackson Pollack circuit) is exactly equivalent to a single battery in series with a single resistor (Fig. 2). This is *Thévenin's Theorem*, with single battery and resistor values V_{Th} and R_{Th}.[1]

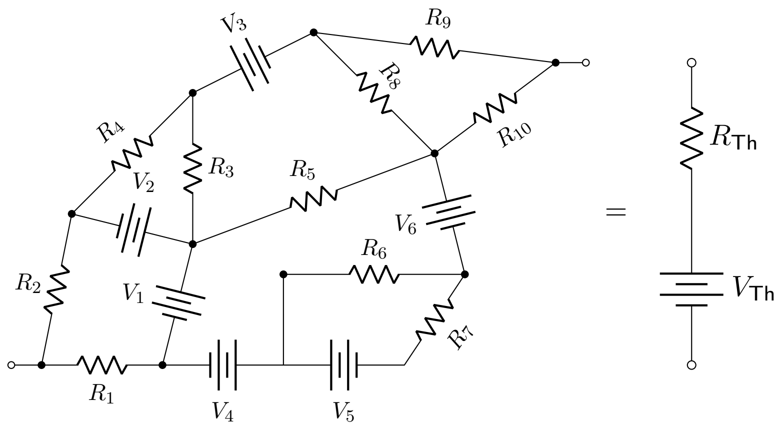


Figure 2: Thévenin equivalent circuit.

How do you figure out the Thévenin equivalent R_{Th} and V_{Th} for a given circuit? Easy! First, you measure the voltage across the Thévenin equivalent circuit *without allowing any current to flow*. The "open-circuit" voltage you get must equal to V_{Th} itself. Next, you connect the two ends of the Thévenin equivalent circuit and measure the resulting current. The "short-circuit" current you get must equal to $V_{\text{Th}}/R_{\text{Th}}$. If the two circuits in Figure 2 behave identically, then the open-circuit voltage and short-circuit current of the real (messy) circuit must satisfy

$$V_{\text{Th}} = V_{\text{open}},$$
$$R_{\text{Th}} = \frac{V_{\text{open}}}{I_{\text{short}}}.$$

Let's apply this method to the voltage divider in Figure 1A (or 1B). The open-circuit voltage and the short-circuit current are

$$V_{\text{open}} = V_{\text{in}} \cdot \frac{R_2}{R_1 + R_2},$$
$$I_{\text{short}} = \frac{V_{\text{in}}}{R_1}.$$

[1] Incidentally, there's another theorem, Norton's theorem, that says you can do the same thing with a current source in parallel with a resistor. Current sources are non-intuitive, don't worry about them for now.

So the Thévenin equivalent circuit is a voltage source

$$V_{\text{Th}} = V_{\text{open}} = V_{\text{in}} \cdot \frac{R_2}{R_1 + R_2}$$

in series with a resistor

$$R_{\text{Th}} = \frac{R_1 R_2}{R_1 + R_2}.$$

(It is not a coincidence that this happens to be the parallel resistance of R_1 and R_2. The reason will become clear later.)

Applying this general result to the particular voltage divider of Figure 1A, we get $V_{\text{Th}} = 5\,\text{V}$ and $R_{\text{Th}} = 5\,\text{k}\Omega$.

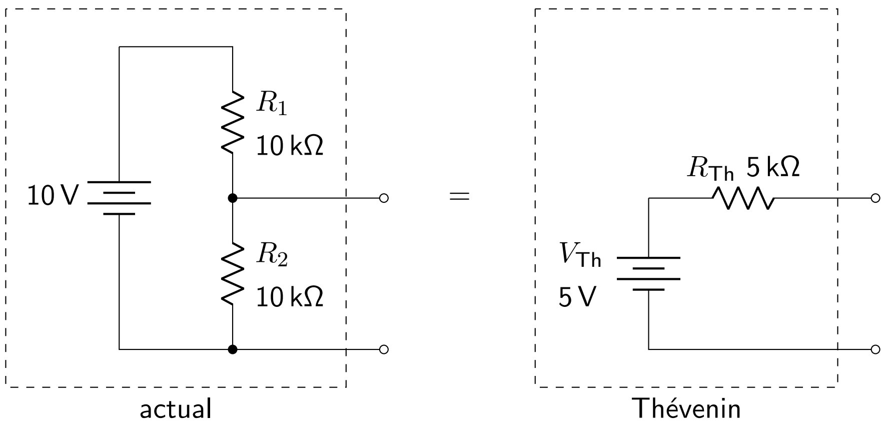


Figure 3: *The Thévenin equivalent of the voltage divider circuit.*

Now, we can analyze the cascaded voltage divider in Figure 1C by combining the Thévenin equivalent (Fig. 3 and the $R_3 R_4$ divider. You will do this in pre-lab hand-in 2.1.

Thévenin Generalized

A lot of work for getting the same result as we got before! But, the power of the Thévenin approach is that it applies to all sorts of sources, not just batteries and voltage dividers. Signal sources (e.g., oscillators, amplifiers, and sensing devices) all have an equivalent internal (Thévenin) resistance. Attaching a load whose resistance is less than or even comparable to the internal resistance will reduce the output considerably. This undesirable reduction of the open-circuit voltage (or signal) by the load is called "circuit loading." Therefore, you should strive to make

$$R_{\text{load}} \gg R_{\text{internal}},$$

because a high-resistance load has little attenuating effect on the source.

We'll see numerous circuit examples in the days ahead. This high-resistance condition ideally characterizes measuring instruments such as voltmeters and oscilloscopes. Speaking of which...

Oscilloscope and Function Generator

Now the fun begins! Voltages don't have to be constant ("dc"), and it's a rare circuit that deals with only steady voltages and currents. The universal tool for looking at signals (time-varying voltages or currents) is the *oscilloscope* ('scope), whose basic function is plotting voltage versus time. And, for benchtop use, you can hardly beat having a good *function generator*, which generates time-varying voltages (sine waves, square waves, pulses, and even "arbitrary" waves).

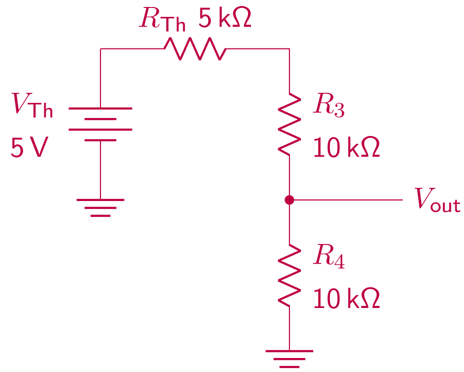
We'll not try to explain all the (sometimes confusing) features of these instruments in this pre-lab. The best way to learn is through "hands-on" — and the lab exercises will give you plenty of opportunities.

Pre-lab 2 Hand-in

Name: ______________________________

Pre-lab 2.1 Figure out the voltage V_{out} in Figure 1C by replacing the divider R_1R_2 with its Thévenin equivalent (Figure 3).

The equivalent circuit looks like this:



This is equivalent to a simple voltage divider made of $R_{\text{Th}} + R_3 = 15\,\text{k}\Omega$ and $R_4 = 10\,\text{k}\Omega$, fed by $V_{\text{Th}} = 5\,\text{V}$. The output voltage is therefore

$$V_{\text{out}} = 5\,\text{V} \cdot \frac{10\,\text{k}\Omega}{15\,\text{k}\Omega + 10\,\text{k}\Omega} = 2\,\text{V}.$$

It is also easy to find V_{X} at the junction between R_{Th} and R_3:

$$V_{\text{X}} = 5\,\text{V} \cdot \frac{20\,\text{k}\Omega}{5\,\text{k}\Omega + 20\,\text{k}\Omega} = 4\,\text{V}.$$

From the last lab:

Pre-lab 2.2 Explain why an ideal current measuring device has zero resistance.

Currents run *through* circuit elements, so we need to place the current meter in *series* with a circuit element in order to measure the current through it. The only thing you can place in series without changing the voltages (and therefore currents) in the circuit would be something with zero resistance, like a piece of wire. Any current meter with a nonzero resistance will drop a voltage across it, disturbing the circuit.

Pre-lab 2.3 **In Lab 1, Figure 5A**, if the $+5\,\text{V}$ input were replaced with a varying voltage (a "signal") represented by a 1 kHz sine wave of 1 V amplitude (i.e., going between -1 V and $+1$ V), and the pot is turned 1/4 of the way up from ground, how does the output voltage at the wiper of the pot do?

If the pot is turned 1/4 of the way up from ground, it forms a "divide-by-four" voltage divider. Thus, its output voltage will be $V_{\text{in}}/4$, regardless of the fact that V_{in} is changing. The output is just a 1 kHz sine wave of 0.25 V amplitude.

Pre-lab 2.4 You want to choose two resistors to build a voltage divider, similar to Figure 1A, to create an output voltage $V_{\text{out}} = 3.3\,\text{V}$ from an applied input voltage $V_{\text{in}} = 5\,\text{V}$. Resistor R_2 is chosen to be $1.0\,\text{k}\Omega$. What is the closest value for R_1 from the available 1% resistor values in the table below? Draw the circuit, with resistor values shown.

100	137	187	255	348	475	649	887
102	140	191	261	357	487	665	909
105	143	196	267	365	499	681	931
107	147	200	274	374	511	698	953
110	150	205	280	383	523	715	976
113	154	210	287	392	536	732	
115	158	215	294	402	549	750	
118	162	221	301	412	562	768	
121	165	226	309	422	576	787	
124	169	232	316	432	590	806	
127	174	237	324	442	604	825	
130	178	243	332	453	619	845	
133	182	249	340	464	634	866	

Table 1: Table of 1% resistor values ("E92"), listed for the decade $100\,\Omega$–$1000\,\Omega$. (For values in other decades simply scale by appropriate powers of ten.)

We'll use the voltage divider equation: $V_{\text{out}} = V_{\text{in}} \cdot \dfrac{R_2}{R_1 + R_2}$. Solving for R_1, we get

$$R_1 = R_2 \cdot \frac{V_{\text{in}} - V_{\text{out}}}{V_{\text{out}}}.$$

Plugging in $V_{\text{in}} = 5\,\text{V}$, $V_{\text{out}} = 3.3\,\text{V}$, and $R_2 = 1\,\text{k}\Omega$ gives $R_1 = 515\,\Omega$. The closest available 1% resistor is $511\,\Omega$.