Pre-lab 1: Voltage, Current, and Resistance

Voltage and Current — Definitions

There are two quantities that we like to keep track of in electronic circuits: voltage and current. These are usually changing with time; otherwise nothing interesting is happening.

Voltage (symbol: V). As discussed in Physics 114, the voltage between two points is the cost in energy (work done) required to move a unit of positive charge from the more negative point (lower potential) to the more positive point (higher potential). Equivalently, it is the energy released when a unit charge moves "downhill" from the higher potential to the lower.

The unit of measure is the *volt*, with voltages usually expressed in volts (V), kilovolts ($1\,\mathrm{kV} = 10^3\,\mathrm{V}$), millivolts ($1\,\mathrm{mV} = 10^{-3}\,\mathrm{V}$), or microvolts ($1\,\mu\mathrm{V} = 10^{-6}\,\mathrm{V}$). A joule (J) of work is needed to move a coulomb (C) of charge through a potential difference of one volt. (The coulomb is the unit of electric charge, and it equals the charge of 6×10^{18} electrons, approximately.)

Current (symbol: I). Current is the rate of flow of electric charge past a point. The unit of measure is the ampere with currents usually expressed in amperes (A), milliamperes ($1\,\mathrm{mA} = 10^{-3}\,\mathrm{A}$), microamperes ($1\,\mu\mathrm{A} = 10^{-6}\,\mathrm{A}$), nanoamperes ($1\,\mathrm{nA} = 10^{-9}\,\mathrm{A}$), or occasionally picoamperes ($1\,\mathrm{pA} = 10^{-12}\,\mathrm{A}$). A current of one ampere equals a flow of one coulomb of charge per second. By convention, current in a circuit is considered to flow from a more positive point to a more negative point, even though the actual electron flow is in the opposite direction.

Important: From these definitions you can see that currents flow *through* things, and voltages are applied (or appear) *across* things. So you've got to say it right: Always refer to current *through* a device or connection in a circuit. Always refer to the voltage *between* two points or *across* two points in a circuit.

However, we do frequently speak of the voltage *at a point* in a circuit. This is always understood to mean the voltage between that point and "ground," a common point in the circuit that everyone seems to know about. Soon you will, too.

Voltage and Current in Circuits

We *generate* voltages by doing work on charges in devices such as batteries (conversion of electrochemical energy), generators (conversion of mechanical energy by magnetic forces), solar cells (photovoltaic conversion of the energy of photons), etc. We *get* currents by placing voltages across things.

At this point you may well wonder how to "see" voltages and currents. The single most useful electronic instrument is the oscilloscope (nickname: 'scope), which allows you to look at voltages (or occasionally currents) in a circuit as a function of time. For voltages or currents that are not rapidly changing, a voltmeter is the instrument of choice. We'll see the latter today, in the form of a "multimeter" (a chameleon — it can measure voltage, current, resistance, and more); next time we'll have fun with the 'scope, looking at voltages that are changing with time ("signals").

Physicists think in terms of electric fields permeating space, with a corresponding electric field potential. This is rarely useful when thinking about circuits. Instead, we think of voltages on wires, and currents through components that connect between wiring. To a good approximation, a wire is an equipotential (it has the same voltage everywhere). A component, for example the humble *resistor* (discussed below), allows a "voltage drop" across its two terminals; for a resistor the corresponding current is

$$I = \frac{V}{R}$$

where R is its resistance (units of ohms, symbol Ω). This is *Ohm's Law*, kinda boring but outrageously useful, as we'll see.

Here are some simple rules about voltage and current:

1. The sum of the currents into a point in a circuit equals the sum of the currents out (conservation of charge). This is sometimes called Kirchhoff's current law ("KCL"). Engineers like to refer to such a point as a *node*. It follows that, for a series circuit (a bunch of two-terminal things all connected end-to-end), the current is the same everywhere.

2. Things hooked in parallel (Fig. 1) have the same voltage across them. Restated, the sum of the "voltage drops" from A to B via one path through a circuit equals the sum by any other route, and is simply the voltage between A and B. Another way to say it is that the sum of the voltage drops around any closed circuit is zero. This is Kirchhoff's voltage law ("KVL").

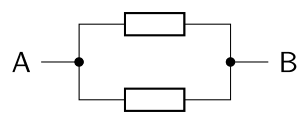


Figure 1: Parallel connection.

3. The power (energy per unit time) consumed by a circuit device is

   $$P = VI$$

   This is simply (energy/charge) × (charge/time). For V in volts and I in amps, P comes out in watts. A watt is a joule per second (1 W = 1 J/s). So, for example, the current flowing through a 60 W light bulb running on 120 V is 0.5 A.

Resistors

It is an interesting fact that the current through a metallic conductor (or other partially conducting material) is proportional to the voltage across it. This is by no means a universal law for all objects. For instance, the current through a neon bulb is a highly nonlinear function of the applied voltage (it is zero up to a critical voltage, at which point it rises dramatically). The same goes for a variety of interesting special devices — diodes, transistors, light bulbs, etc. (If you are interested in understanding why metallic conductors behave this way, read sections 4.4–4.5 in Purcell and Morin's splendid text *Electricity and Magnetism*).

A resistor is made out of some conducting stuff (carbon, or a thin metal or carbon film, or wire of poor conductivity), with a wire or contacts at each end. It is characterized by its resistance:

$$R = \frac{V}{I}$$

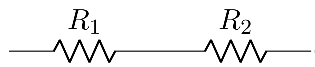
(Ohm's Law), where R is in ohms for V in volts and I in amperes. Typical resistors of the most frequently used types (metal-oxide film, metal film, or carbon film) come in values from 1 ohm (1 Ω) to about 10 megohms (10 MΩ).

Resistors are also characterized by how much power they can safely dissipate (the most commonly used ones are rated at 1/4 or 1/8 watt), their physical size, and by other parameters such as tolerance (accuracy), temperature coefficient, noise, voltage coefficient (the extent to which R depends on applied V), stability with time, inductance, etc.

Roughly speaking, resistors are used to convert a voltage to a current, and vice versa. This may sound awfully trite, but you will soon see what we mean.

Resistors in Series and in Parallel

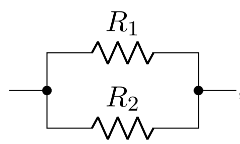
From the definition of R, some simple results follow:

1. The resistance of two resistors in series, as in 

, is

   $$R = R_1 + R_2. \tag{1}$$

   By putting resistors in series, you always get a *larger* resistor.

2. The resistance of two resistors in parallel, as in 

, is

   $$R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}. \tag{2}$$

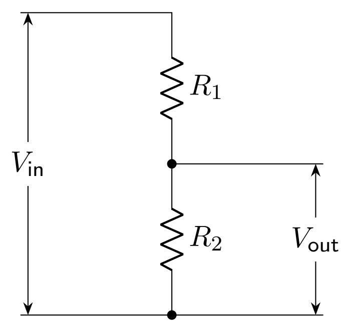
   By putting resistors in parallel, you always get a *smaller* resistor.

Voltage Divider

Nearly all electronic circuits accept some sort of applied *input* (usually a voltage) and produce some sort of corresponding *output* (which again is often a voltage). For example, an audio amplifier might produce a (varying) output voltage that is 100 times as large as a (similarly varying) input voltage. When describing such an amplifier, we imagine measuring the output voltage for a given applied input voltage. Engineers speak of the *transfer function* $\mathbf{H}$, the ratio of (measured) output divided by (applied) input; for the audio amplifier above, $\mathbf{H}$ is simply a constant ($\mathbf{H} = 100$). We'll get to amplifiers soon enough. However, with only resistors we can already look at a very important circuit fragment, the *voltage divider* (which you might call a "de-amplifier").

The voltage divider is one of the most widespread electronic circuit fragments. Show us any real-life circuit and we'll show you half a dozen voltage dividers. To put it very simply, a voltage divider is a circuit that, given a certain voltage input, produces a predictable fraction of the input voltage as the output voltage. The simplest voltage divider is shown in Figure 2.

An important word of explanation: When engineers draw a circuit like this, it's generally assumed that the V_{in} on the left is a voltage that you are applying to the circuit, and that the V_{out} on the right is the resulting output voltage, produced by the circuit, that you are measuring (or at least are interested in). You are supposed to know all this (a) because of the convention that signals generally flow from left to right, (b) from the suggestive names ("in," "out") of the signals, and (c) from familiarity with circuits like this. This may be confusing at first, but with time it becomes easy.



$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{R_2}{R_1 + R_2}$$

Figure 2: Voltage divider. An applied voltage V_{in} results in a (smaller) output voltage V_{out}.

What is V_{out}? Well, the current (same everywhere, assuming no "load" on the output; i.e., nothing connected across the output) is

$$I = \frac{V_{\text{in}}}{R_1 + R_2}$$

(We've used Ohm's law and the series law.) Then, for R_2,

$$V_{\text{out}} = IR_2 = \frac{R_2}{R_1 + R_2} V_{\text{in}}$$

Note that the output voltage is always less than (or equal to) the input voltage; that's why it's called a divider.

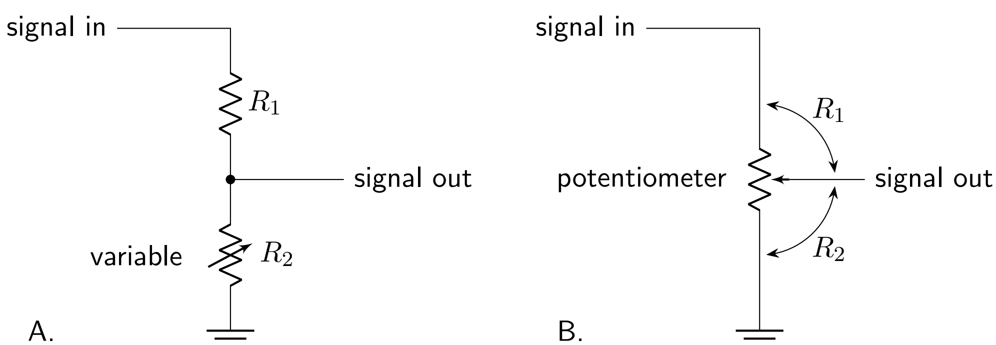


Figure 3: An adjustable voltage divider can be made from a fixed and a variable resistor (A), or from a potentiometer (B).

Voltage dividers are often used in circuits to generate a particular voltage from a larger fixed (or varying) voltage. For instance, if V_{in} is a varying voltage and R_2 is an adjustable resistor (Fig. 3A), you have a "volume control"; more simply, the combination of R_1 and R_2 can be made from a single variable resistor, or *potentiometer* (Fig. 3B).

The humble voltage divider is even more useful, though, as a way of *thinking* about a circuit: the input voltage and upper resistance might represent the output of an amplifier, say, and the lower resistance might represent the input of the following stage. In this case the voltage-divider equation tells you how much signal gets to the input of that last stage. This will all become clearer after you know about a remarkable fact (Thévenin's theorem) that we'll see in the second class/lab.

Pre-lab 1 Hand-in

Name: ______________________________

Pre-lab 1.1 You have a $5\,\mathrm{k}\Omega$ resistor and a $10\,\mathrm{k}\Omega$ resistor. What is their combined resistance (a) in series and (b) in parallel?

(a) Series: resistances add. $R_{\text{series}} = 5\,\mathrm{k}\Omega + 10\,\mathrm{k}\Omega = 15\,\mathrm{k}\Omega$.

(b) Parallel: resistances add inversely, i.e., $\dfrac{1}{R_{\text{parallel}}} = \dfrac{1}{5\,\mathrm{k}\Omega} + \dfrac{1}{10\,\mathrm{k}\Omega}$, so

$$R_{\text{parallel}} = \frac{5\,\mathrm{k}\Omega \cdot 10\,\mathrm{k}\Omega}{5\,\mathrm{k}\Omega + 10\,\mathrm{k}\Omega} = 3.3\,\mathrm{k}\Omega.$$

Pre-lab 1.2 If you place a $1\,\Omega$ resistor across a $12\,\mathrm{V}$ car battery, how much power will it dissipate?

We will use $P = VI = \dfrac{V^2}{R}$ to solve this.

$$P = \frac{(12\,\mathrm{V})^2}{1\,\Omega} = 144\,\mathrm{W}.$$

That is a lot of power, and our usual quarter-Watt resistors will not be able to handle it! (For context, a human body dissipates about 100 Watts of heat at all times, and a resistor is much smaller than your body.)

Pre-lab 1.3 Prove the formulas for series and parallel resistors, given in equations (1) and (2), using Ohm's law and Kirchhoff's law.

(a) Series: By Kirchhoff's laws, current through both resistors is the same: $I_1 = I_2 = I$. The total voltage across both resistors V is the sum of the voltages across each resistor: $V = V_1 + V_2$. Using Ohm's Law $V = IR$ to substitute out voltages:

$$V = IR_{\text{series}} = V_1 + V_2 = IR_1 + IR_2.$$

Solving for R_{series} gives the desired result:

$$R_{\text{series}} = R_1 + R_2.$$

(b) Parallel: By Kirchhoff's Laws, the voltage across both resistors is the same: $V_1 = V_2 = V$. The total current through both resistors is the sum of the currents through each resistor: $I = I_1 + I_2$. Using Ohm's Law, this time in the form $I = V/R$:

$$I = \frac{V}{R_{\text{parallel}}} = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2}.$$

Thus,

$$\frac{1}{R_{\text{parallel}}} = \frac{1}{R_1} + \frac{1}{R_2}.$$