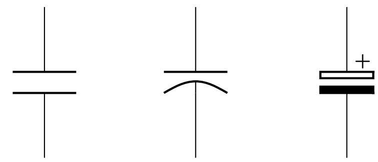
Capacitors in the Time Domain

Today we discuss the behavior of RC circuits in the time domain (i.e. what you see on the scope). RC circuits can, under some circumstances, be used to do calculus on a signal, no calculator required!

Outline

1 Capacitors

$$Q = CV$$



Q is the charge across the capacitor: one plate will have charge $+Q$ and the other plate will have charge $-Q$. V is the voltage across the capactor. C is the *capacitance*, which is a measure of the ratio of charge/volt that the capacitor can support. It is derived entirely from the geometry and materials of the capacitor. The unit for capacitance is a Farad, $F = [C]/[V]$.

Capacitance measures how much charge accumulates per volt across the device. A large C means there will be a lot of charge for a given voltage, which will require a lot of current to charge. Conversely, a small C will require little current to charge to a given voltage.

Some capacitors are polarized, meaning charge can only flow through them in one direction. It is important to connect them in the right direction because their power threshold is usually pretty low and you will break down the dielectric material inside if you exceed it in the wrong direction. The middle and far right capacitors in the figure above are polarized. The longer leg of the device will be the positive leg.

1.1 Reading Capacitor Values

Although the units of capacitors are Farads, these are much too large to be practical (like Coulombs). Instead we usually see capacitors in units of picofarads (pF, 10^{12}), nanofarads (nF, 10^{-9}), or microfarads (μF, 10^{-6}). There are a few important quantities to consider when selecting a capacitor:

1. There are different types of capacitors that suit different purposes (radial/axial/plate, electrolytic, dielectric, etc.). They have different qualities that are more usefully summarized by the ratings described below.
2. Capacitance is often written as a three-digit code, where the first two digits are the first two digits and the third digit says how many zeros to add on the end. The result is then in units of pF.
3. Capacitors come in a much wider range of tolerances than resistors. We are most likely to see 5-10% tolerance capacitors, but keep an eye out for a tolerance code, usually a letter at the end of the three-digit capacitance code.
4. Maximum voltage rating has its own special code that will appear before the three-digit capacitance code. Exceeding the voltage rating on your capacitor can lead to anything from silent failure to explosion.

This is not an exhaustive description of capacitor codes. You can find more information and all the codes here: https://www.raypcb.com/how-to-read-capacitor/

1.2 Adding Capacitors

Capacitors in parallel add directly: $C_{||} = C_1 + C_2$

Capacitors in series add in inverse: $C_{series} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)$

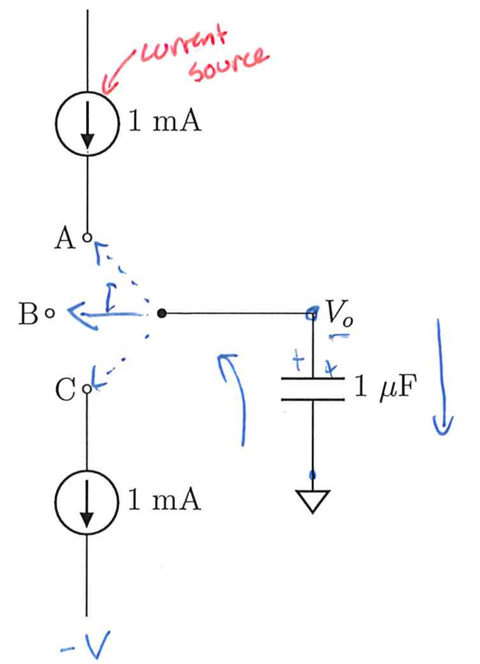
3 Charging and Discharging Capacitors

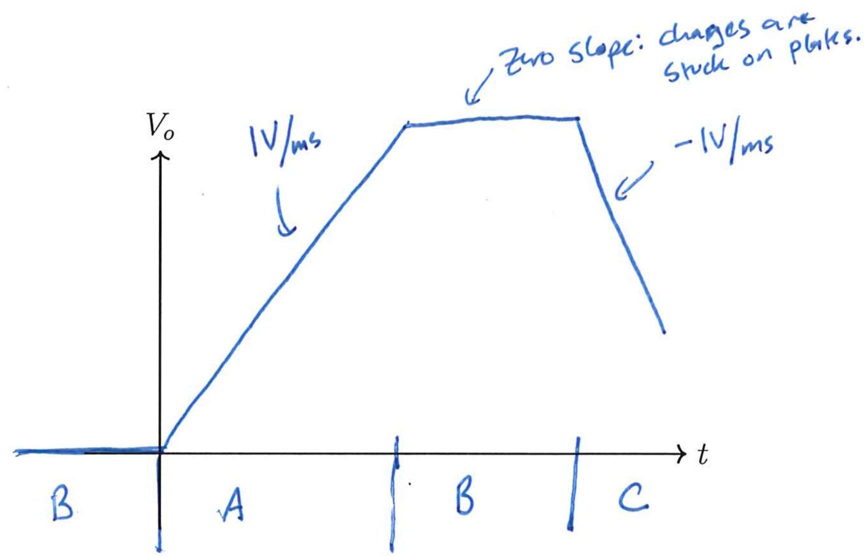
$$Q = CV$$

$$\frac{dQ}{dt} = I = C\frac{dV}{dt}$$

$$\frac{dV}{dt} = \frac{I}{C} = \frac{1\,\text{mA}}{1\,\mu\text{F}} = \frac{1}{1\text{m}} = 1000\ \frac{\text{V}}{\text{s}}$$
$$1\ \text{V/ms}$$

3.1 Constant Current





We rarely have a current source. Usually our sources are constant-voltage.

3.2 RC Charging

Charging:

$$V_o = V_+ \left(1 - e^{\frac{-t}{RC}}\right)$$

$\tau = RC$

at $t=0$

+V

I

0V

0V

$V = IR$

$I = \frac{V_+}{R}$

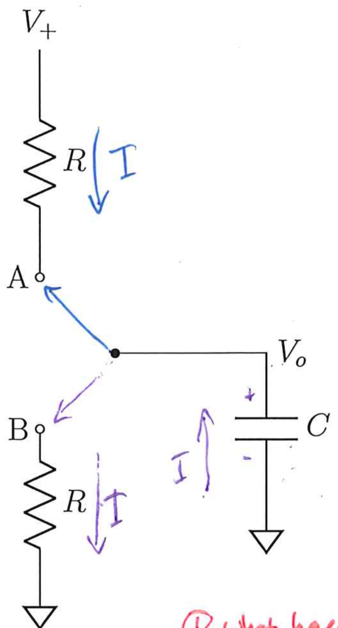
Ohm's

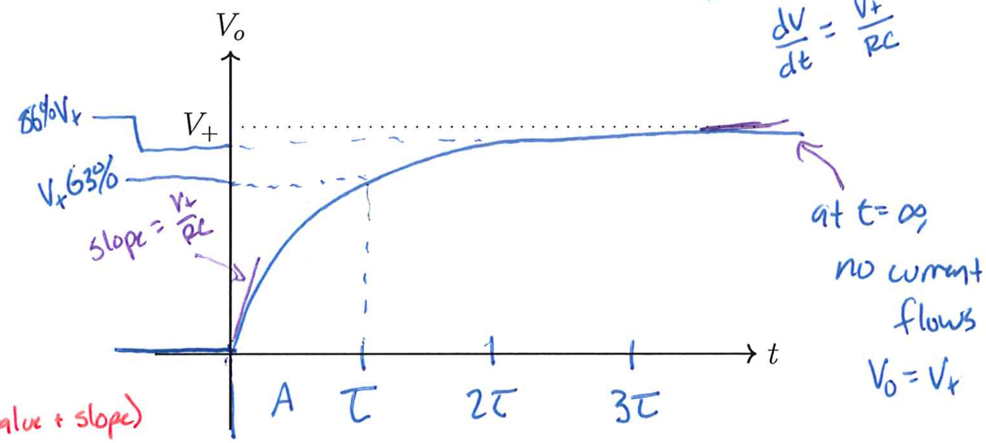
$I = C\frac{dv}{dt}$

capacitance

$\frac{V_+}{R} = C\frac{dV}{dt}$

$\frac{dV}{dt} = \frac{V_+}{RC}$





① what happens (value + slope) at $t=0$?

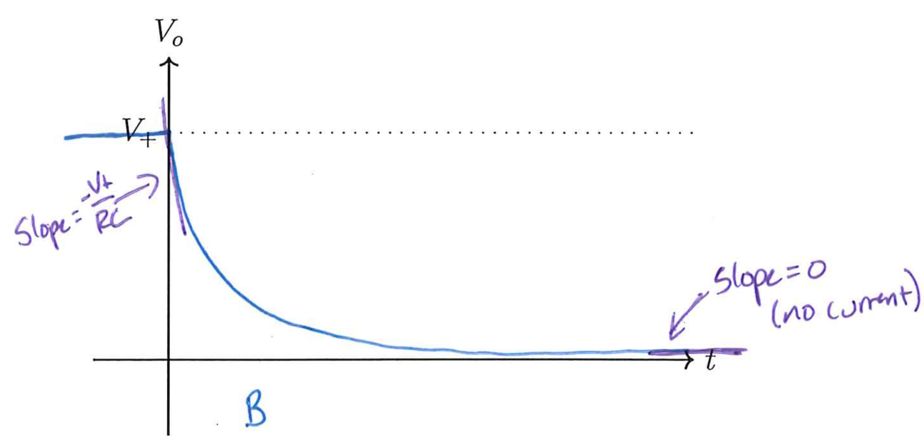
② what happens (value + slope) at $t=\infty$?

③ connect with appropriate exponential

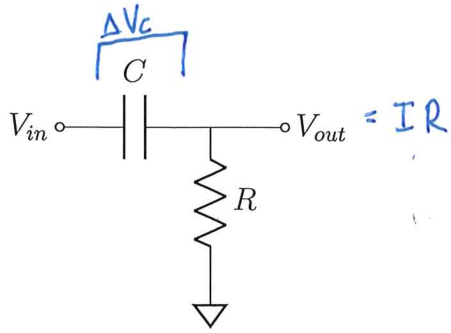
τ	2τ	3τ	4τ	5τ
0.63	0.86	0.95	0.98	0.99

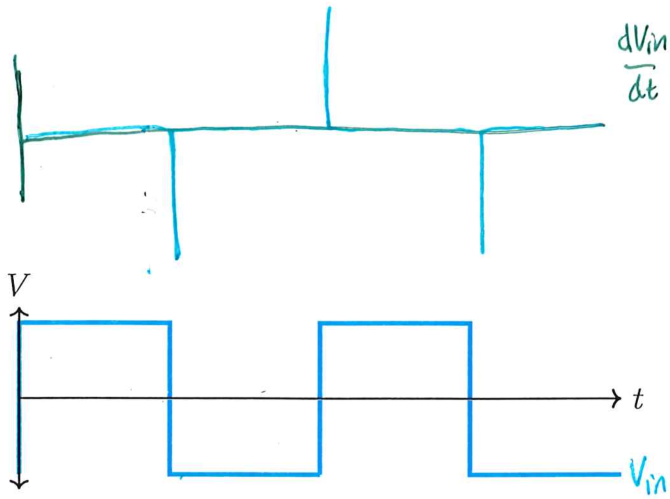
Disharging:

$$V_o = V_+ \left(e^{\frac{-t}{RC}}\right)$$



4 Differentiator





$$I_c = C\frac{dV_c}{dt}$$

$$V_{out} = IR = C\frac{dV_c}{dt}R = RC\frac{dV_c}{dt}$$

$$V_{in} = V_c + V_R$$

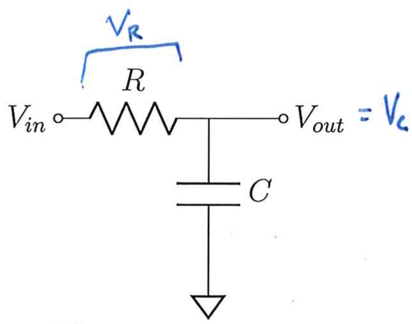
$$V_c = V_{in} - V_R$$

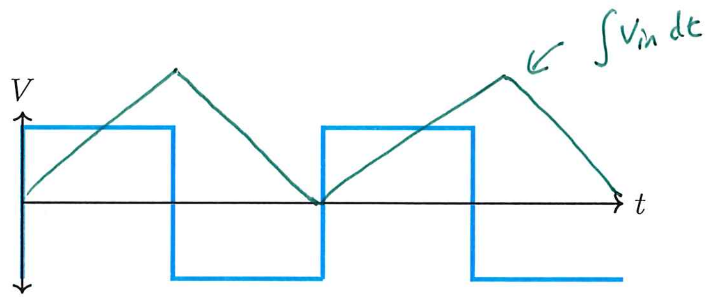
$$V_{out} = RC\frac{d}{dt}(V_{in} - V_R)$$

If $V_R \ll V_{in}$, then this circuit works as a differentiator!

$V_{out} \ll V_{in}$

5 Integrator





$$V_{out} = V_c$$

$$I_c = C\frac{dV_c}{dt}$$

$$\int \frac{I_c}{C}dt = \int dV_c$$

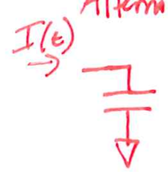
$$V_c = V_{out} = \frac{1}{C}\int I_c dt \quad \text{but } \overset{\text{KCL}}{I_c = I_R} \overset{\text{ohm}}{=} \frac{V_R}{R}$$

$$V_{out} = \frac{1}{C}\int \frac{V_R}{R}dt = \frac{1}{RC}\int V_R\, dt \quad \text{but } \overset{\text{KVL}}{V_R = V_{in} - V_c}$$

$$V_{out} = \frac{1}{RC}\int (V_{in} - V_c)\, dt$$

As long as $V_{out} \ll V_{in}$, works as an integrator.

Alternate understanding



If signal is $I(t)$, then $V_{out} = \frac{1}{C}\int I dt$ is a perfect integrator. To turn $I(t)$ into $V(t)$, insert a resistor: