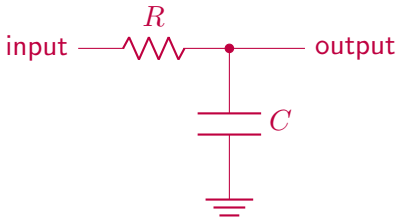
Lab 5: Capacitors — Low- and High-Pass Filters

RC Low-pass Filter — Design

(a) We will start by building a low-pass RC filter. Draw a generic **low-pass RC filter** below, without component values yet. Where is the input, and where is the output?



(b) Write down the resistance R_R of the resistor and the reactance X_C of the capacitor as a function of frequency. Which is larger at high frequencies ($\omega \to \infty$)? At low frequencies ($\omega \to 0$)?

$$R_R = R$$

$$X_C = \frac{1}{j\omega C}$$

- At high frequency, R_R is larger.
- At low frequency, X_C is larger.

(c) You can think of the filter as a voltage divider, with the capacitor's reactance replacing one of the resistors. Using this conceptualization, circle the correct answers:

- At high frequencies ($\omega \to \infty$), most of the input voltage drops across

  (a) the resistor (b) the capacitor.

- At low frequencies ($\omega \to 0$), most of the input voltage drops across

  (c) the resistor (d) the capacitor.

(d) Set $R = 10\,\text{k}\Omega$ and $C = 10\,\text{nF}$. What is the crossover frequency f_c **in Hz**?

$$f_c = \frac{1}{2\pi RC} = 1.59\,\text{kHz}.$$

(e) By what factor is the signal attenuated at the crossover frequency? How many decibels is this?

A factor $1/\sqrt{2} \approx 0.7$, which corresponds to $-3\,\text{dB}$.

RC Low-pass and High-pass filters: compare

(a) Build the RC low-pass filter with $R = 10\,\text{k}\Omega$ and $C = 10\,\text{nF}$. Drive it with a sine wave from the function generator. Use CH1 and CH2 of the scope to observe the input and output waveforms.

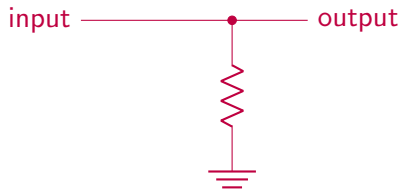
Go to the crossover frequency you've found on the last page, and check the attenuation factor against your prediction. You may use the scope's Measure menu, in which you can find the function to measure and display the signal amplitude for each channel.

(b) "At very low frequencies, a capacitor can be considered an open circuit," and "At very high frequencies, a capacitor can be considered a short." Explain this statement in terms of what you have observed.

At very low frequencies, the impedance $1/j\omega C$ of a capacitor approaches infinity. This is equivalent to removing the capacitor from the circuit, which turns the LPF to



This would make output = input, as you must have observed. At very high frequencies, the impedance of a capacitor approaches zero. This is equivalent to replacing the capacitor with a piece of wire. This turns the HPF into



This makes output = input, as you must have observed.

(c) Drive the circuit with a 10 kHz sine wave. Then, use the offset option on the function generator to add a 1 V offset to the waveform. Draw the input and output waveforms below, and explain the output in terms of the frequency response of the filter.[1]

You should see identical waveforms at input and output *except* for the 1 V DC offset. The input is now centered at +1 V, but the output is still centered at 0 V. This is because the "DC" offset has a frequency of 0 Hz and cannot pass the HPF.

[1] When we use a high-pass filter to block out DC, we often say we have used a *blocking capacitor*, or that we have *AC coupled* the signal. These are just different terms for high-pass filtering (often with a very low cutoff frequency) that is used specifically to block the DC component of a signal.

Time Permitting: RC Band-pass Filter

Sometimes, a physical signal of interest lies in between low- and high-frequency noises. A band-pass filter passes a "band" of frequencies of interest while attenuating those that are too high or low.

(a) Figure **??** shows a possible design of RC band-pass filters.

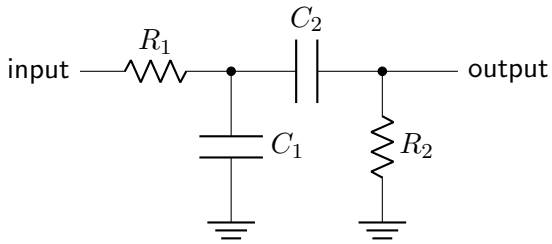


Figure 1: An RC band-pass filter.

We assume the input signal (what we want to transmit) is 1 kHz sine waves.

Let's consider the left-hand circuit, and choose the component values R_1, R_2, C_1, and C_2. We want to satisfy the following design critaria:

1. the crossover frequencies for the low- and high-pass filters are at least one decade from the center frequency of 1 kHz;
2. the input signal comes form a function generator with 1 kΩ source impedance.
3. the output of your filter drives a circuit of 1 MΩ input impedance.

Each time you connect two circuits, make sure that the source impedance is smaller than the load impedance by at least a factor 10.

$R_1 = 10\,\text{k}\Omega$ $C_1 = 1.6\,\text{nF}$

$R_2 = 100\,\text{k}\Omega$ $C_2 = 16\,\text{nF}$

First, we choose resistors based on input and output impedance of each stage. For any filter, which is like a (complex) voltage divider, $Z_{in} = Z_R + Z_C$ (series addition), and $Z_{out} = Z_R || Z_C$ (parallel addition). The impedance of the capacitor changes with frequency, but if we think about the *worst case* (that is, the lowest possible input impedance and the highest possible output impedance), we find that the input impedance of a filter is always *at least* R, and the output impedance is *at most* R. So we can just choose R to increase by 10 in each stage. The circuit is driven by a function generator with output impedance $R_s = 1\,\text{k}\Omega$, so $R_1 = 10\,\text{k}\Omega$ so that it is 10 times larger than the source impedance of the function generator. Second, we choose $R_2 = 100\,\text{k}\Omega$ so that it is 10 times smaller than the input impedance of the next circuit. Conveniently, this means that the first filter is also able to drive the second filter. We then choose the capacitor values so that the crossover frequencies of the LPF (R_1 and C_1) and the HPF (R_2 and C_2) to be 10 kHz and 100 Hz, respectively.

(b) Build the circuit in real life. Does it behave as you would like it to?

Hopefully it did.

(c) Does the order of the two stages matter? What happens if we swapped low-pass and high-pass filter, as shown below.

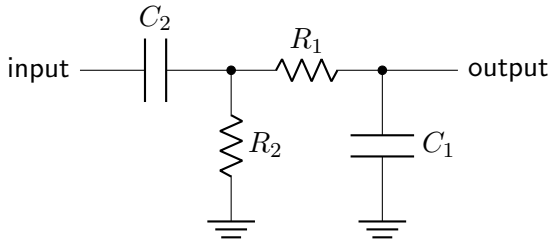


Figure 2: RC band-pass filter, swapped.

Predict what might happen, and test your prediction in the actual circuit.

If you flipped the LPF and the HPF, you probably saw the output to be much smaller than before.

The reason is that we used $R_1 = 10\,\text{k}\Omega$ and $R_2 = 100\,\text{k}\Omega$. That means the output impedance of the LPF is larger than the input impedance of the HPF. This is in violation of our cardinal rule: **"Always drive a larger impedance with a smaller impedance by at least a factor 10."**

Note that the circuit before swapping (LPF before HPF) satisfied the rule.

So, is it impossible to make order-reversed circuit work? Not at all! What we have to do is to change the resistor values so that R_2 is smaller than R_1, then recalculate C_1 and C_2 to keep the crossover frequencies unchanged. Given the conditions for the function generator and the next circuit, our choices are limited to

$R_1 = 100\,\text{k}\Omega$ $C_1 = 160\,\text{pF}$

$R_2 = 10\,\text{k}\Omega$ $C_2 = 160\,\text{nF}$