Tutorial: Rotating Frames of Reference

0. Fictitious forces, and changing reference frames

The following is a quick review for your reference!

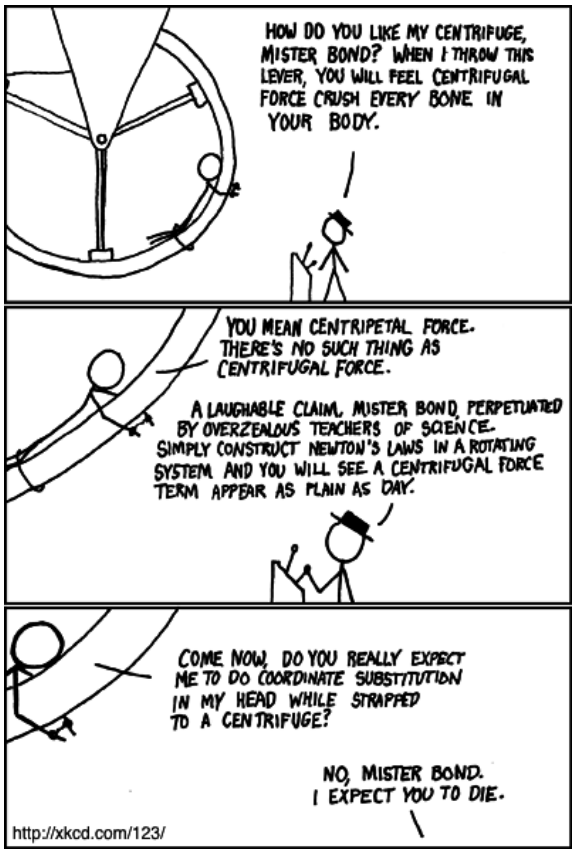
As we saw in class, when dealing with motion involving *rotation*, e.g. a rotating platform like a merry-go-round, we can apply Newton's laws of motion either in the *ground frame* (what a person standing next to the merry-go-round observes), or in the *rotating frame* (what a person standing on the merry-go-round observes.)

If we put the origin of both coordinate systems at the same point, and the rotating frame is described by angular velocity vector $\vec{\Omega}$, then the velocity of a moving object in the ground frame (coordinates (x', y', z')) is related to that in the rotating frame (coordinates (x, y, z)) by the equation

$$\vec{v'} = \vec{v} + \vec{\Omega} \times \vec{r}. \tag{1}$$

When we use Newton's laws in the *rotating frame*, we must include up to two *fictitious forces* that appear as a result of the rotation:

$$\begin{cases} \vec{F}_{\text{cf}} = m(\vec{\Omega} \times \vec{r}) \times \vec{\Omega} & \text{(centrifugal force)} \\ \vec{F}_{\text{cor}} = 2m\dot{\vec{r}} \times \vec{\Omega} & \text{(Coriolis force)} \end{cases} \tag{2}$$



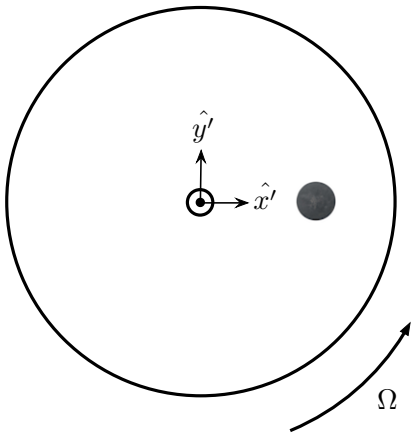
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I. Consistency between frames of reference

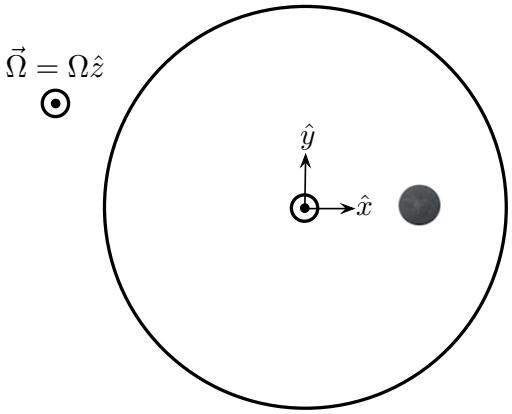
In the following we will ignore gravity and normal forces, only considering motion in the plane of the platform.

A level, **frictionless** platform rotates *counter-clockwise* at a constant rate Ω relative to the ground frame. For our first example, a puck of mass m is placed **at rest in the ground frame** at position $R\hat{x}'$, where our origin is at the center of the platform. In the ground frame, the puck remains at rest while the platform spins (since there are no forces acting on it in that frame!)

Top view ***in ground (inertial) frame***



Top view ***in platform (rotating) frame***



A. Now consider the point-of-view of an observer who is stationary in the *rotating* frame. What does the motion of the puck look like to them? Sketch its path in the rotating coordinate system on the diagram. (Hint: if I use a ruler to measure the distance of the puck from the center in the ground frame, and then in the rotating frame, will I find different answers, or the same?)

B. The puck starts at rest in the inertial (ground) frame, so $\vec{v'} = 0$. What is its initial velocity $\vec{v}$ in the *rotating* frame?

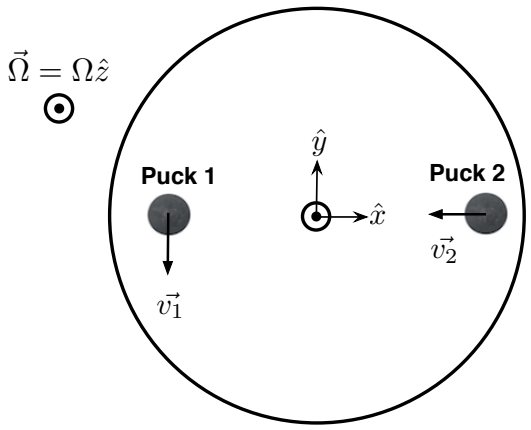
C. Using the initial speed $\vec{v}$ you just found, calculate the *magnitude* and *direction* of the centrifugal and Coriolis forces acting on the puck at its starting position of $\vec{r} = R\hat{x}$. What is the net force? Does this seem consistent with the force needed to move the puck along the path you sketched in part B?

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II. Motion on a rotating platform

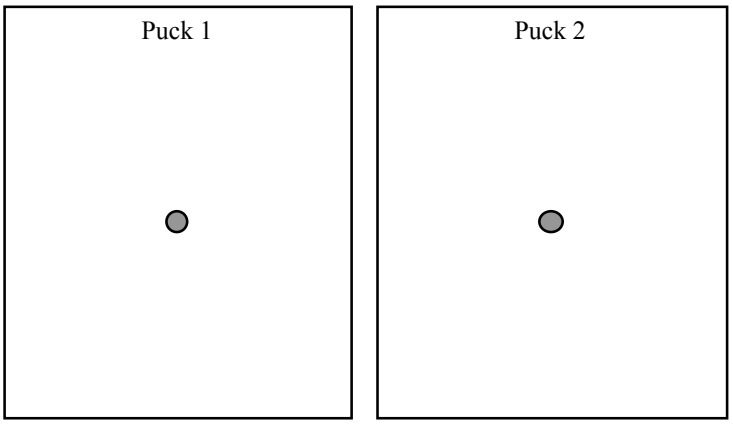
Now consider two pucks with initial velocities $\vec{v_1}, \vec{v_2}$ as measured in the *rotating* frame.

Top view ***in platform (rotating) frame***



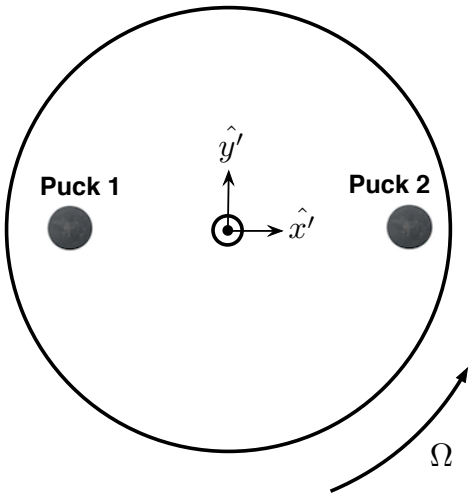
A. For the instant shown in the top-view diagram (*left*), draw separate free-body diagrams **in the boxes below** for the two pucks, including all non-zero fictitious forces. (Again, we are drawing this from the top-down view, so don't worry about gravitational or normal forces, and there is still no friction.)

Free-body diagrams *in platform frame*



B. Will puck 2, which is initially moving in the $-\hat{x}$ direction in the platform frame, ever reach the exact center of the platform? Use your free-body diagram from part A to support your answer. **On the diagram above**, sketch a qualitatively correct path followed by each puck as seen in the *rotating* frame.

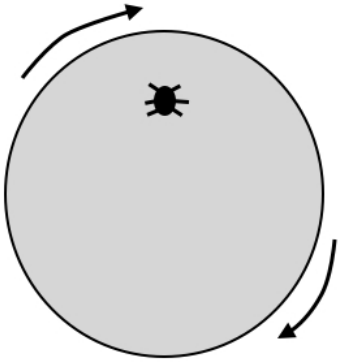
Top view ***in ground (inertial) frame***



C. Using the relationship $\vec{v'} = \vec{v} + \Omega \times \vec{r}$, sketch the initial velocities $\vec{v_1'}, \vec{v_2'}$ on the ground-frame diagram to the right. Based on these initial velocities, also sketch a qualitatively correct path for the two pucks as seen in the ground frame.

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III. Adding friction



Now a bug is standing on the surface of another turntable, rotating **clockwise** (so $\vec{\Omega} = -\Omega\hat{z}$.) Unlike the pucks, the bug experiences a **frictional force**, with coefficient of static friction μ_S. (This means that we can't ignore normal forces anymore! Assume the usual gravitational acceleration g directed into the page.)

A. The bug begins at rest at a distance r_0 from the center of the turntable, directly above it. Draw a free-body diagram *in the rotating frame*. Given that the bug is *not* sliding off as the turntable spins, what is the largest possible value of r_0, in terms of Ω, μ_S and g?

B. Now the bug starts to walk radially out towards the edge of the platform, at a constant speed v_0 (starting from r_0.) Sketch a new free-body diagram, again in the rotating frame. Now at what value of r will the bug start to slip?

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