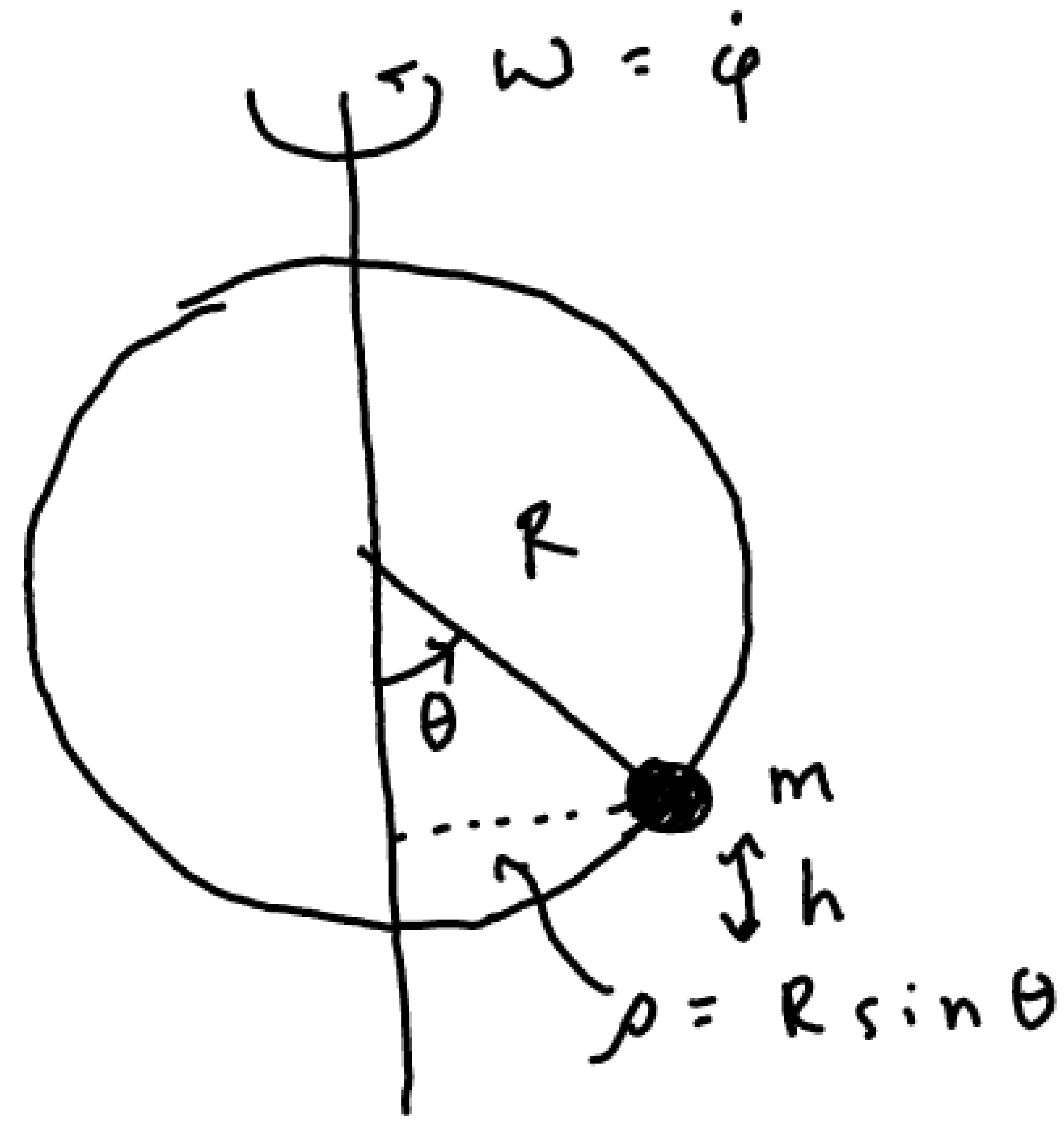


This system is

<u>non-relativistic</u>

because the hoop is spinning,
which gives the bead centripedal acceleration.

First, write the Lagrangian in inertial coordinates.
We can fully specify the motion with θ.

motion is purely rotational on two axes: θ, φ

$$T = \frac{1}{2} m R^2 \dot\theta^2 + \frac{1}{2} m \rho^2 \dot\varphi^2$$

$$= \frac{1}{2} m R^2 \left(\dot\theta^2 + \omega^2 \sin^2\theta \right) \qquad \text{using } \dot\varphi = \omega$$

$$\mathcal{U} = mgh = mg(R - R\cos\theta) \qquad \text{ignore constant term.}$$

$$\mathcal{L} = T - \mathcal{U} = \frac{1}{2} m R^2 \dot\theta^2 + mgR\cos\theta$$

Solve Euler-Lagrange in θ

$$\frac{\partial \mathcal{L}}{\partial \theta} = mR^2 \omega^2 \sin\theta \cos\theta - mgR\sin\theta$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\theta}} = mR^2 \ddot{\theta}$$

$$\cancel{m} R^{\cancel{2}} \omega^2 \sin\theta \cos\theta - \cancel{m}g\cancel{R}\sin\theta = \cancel{m} R^{\cancel{2}} \ddot{\theta}$$

$$\ddot{\theta} = \omega^2 \sin\theta\cos\theta - \frac{g}{R}\sin\theta$$

$$\ddot{\theta} = \left(\omega^2 \cos\theta - \frac{g}{R}\right)\sin\theta$$

which we cannot solve analytically

<u>BUT</u> we can look at behavior in some useful limiting cases.

Equilibrium Positions

Any point where if our object starts there at rest, then it will remain at rest.

In mathematical terms, $\frac{d\dot{\theta}}{dt} = \ddot{\theta} = 0$.

Then $\left(w^2 \cos\theta - \frac{g}{R}\right)\sin\theta = 0$

Two eq. points: $\sin\theta = 0 \rightarrow \theta = 0$ or $\pi/2$

bottom or top of hoop.

$w^2 \cos\theta_0 = g/R$

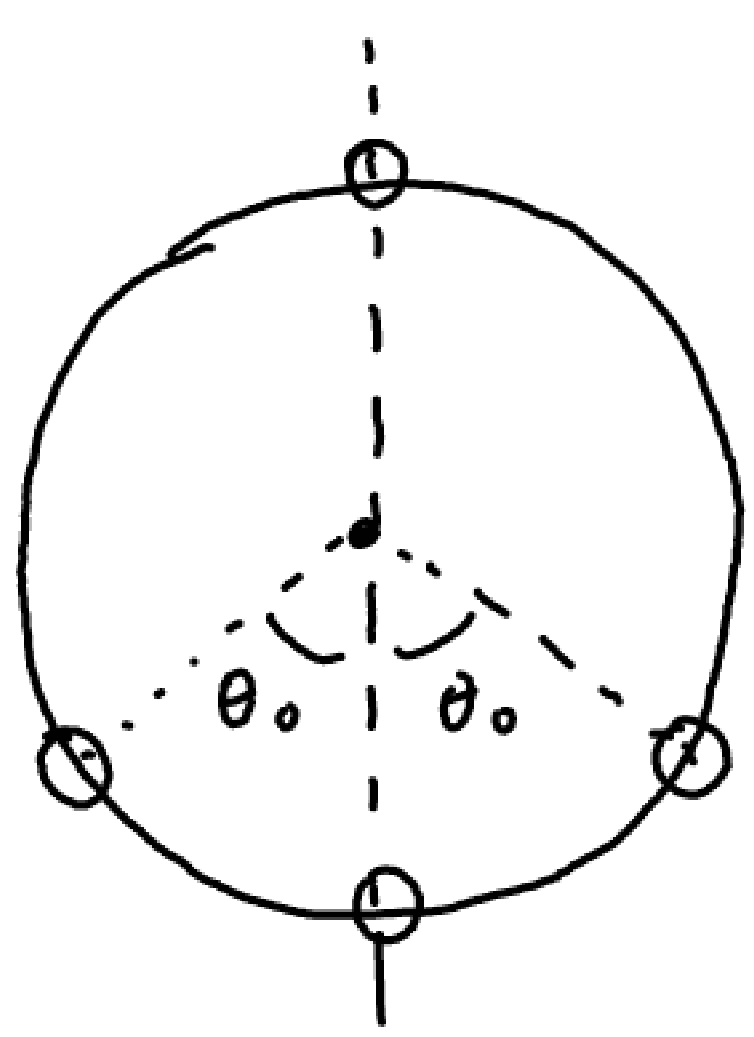
$\cos\theta_0 = \frac{g}{w^2 R}$ $\leftarrow$ $|\cos\theta| \leq 1$

so $\frac{g}{w^2 R} \leq 1$

$\theta_0 = \pm \cos^{-1}\left(\frac{g}{w^2 R}\right)$

$w^2 > g/R$

condition for second set of eq. points.



Four possible eq. points for high speed $w^2 > g/R$

Only two for low speed $w^2 < g/R$