# Symmetric Top precession

$$\lambda_1 \dot{\omega}_1 - (\lambda_2 - \lambda_3)\omega_2\omega_3 = 0$$

$$\lambda_2 \dot{\omega}_2 - (\lambda_3 - \lambda_1)\omega_3\omega_1 = 0$$

$$\lambda_3 \dot{\omega}_3 - (\lambda_1 - \lambda_2)\omega_1\omega_2 = 0$$

$\lambda_1 = \lambda_2 \neq \lambda_3 \rightarrow \lambda_3 \dot{\omega}_3 = 0$

$\omega_3$ is constant

BUT $\omega_1, \omega_2$ may evolve in time

asymmetric top: had to take $\omega_1, \omega_2$ small

symmetric top: not limited

$\lambda_1 = \lambda_2$:

$$\dot{\omega}_1 = \frac{\lambda_1 - \lambda_3}{\lambda_1}\omega_3\ \omega_2 = \Omega_b \omega_2$$

$$\dot{\omega}_2 = -\frac{\lambda_1 - \lambda_3}{\lambda_1}\omega_3\ \omega_1 = -\Omega_b \omega_1$$

for $\Omega_b = \frac{\lambda_1 - \lambda_3}{\lambda_1}$

Use the trick from HW7 to solve:

$$\eta = w_1 + i w_2$$

$$\dot{\eta} = \dot{w}_1 + i \dot{w}_2$$

$$\ddot{\eta} = \ddot{w}_1 + i \ddot{w}_2$$

$$\dot{w}_1 + i\dot{w}_2 = \Omega_b w_2 - i\Omega_b w_1$$

$$= -i\Omega_b (w_1 + i w_2)$$

$$\dot{\eta} = -i\Omega_b \eta$$

$$\eta(t) = \eta_0 e^{-i\Omega_b t}$$

Let $w_2 = 0$ at $t = 0 \rightarrow \eta_0 = w_0$

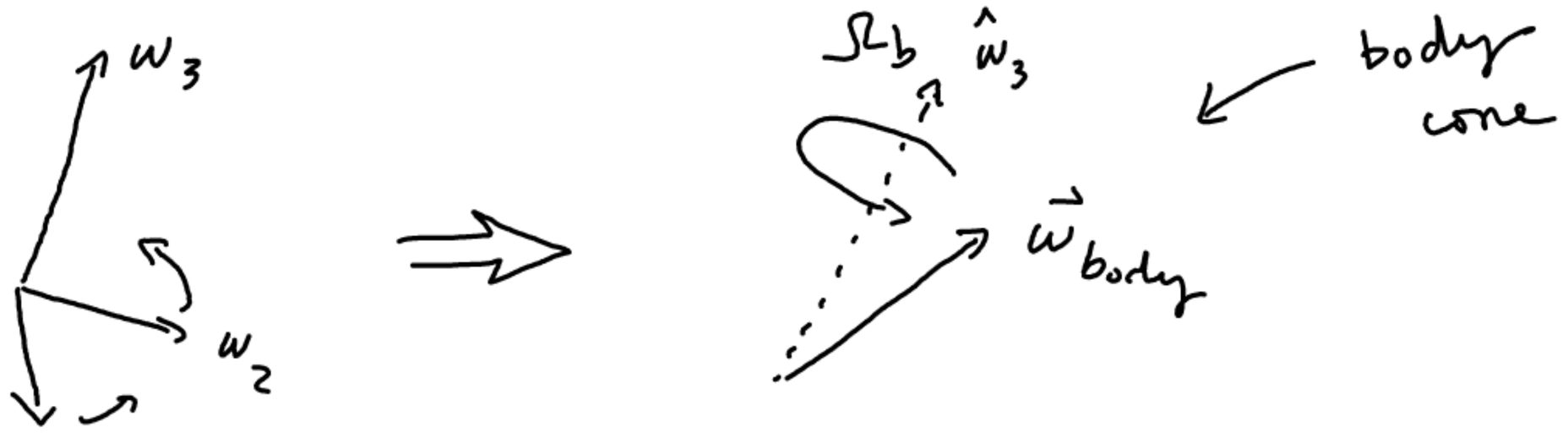
$$\vec{w}(t) = (w_0 \cos\Omega_b t, -w_0 \sin\Omega_b t, w_3)$$

$w_3$ const

$w_1, w_2$ circle at frequency $\Omega_b$

const. frequency rotation of $\vec{w}$ is called <u>precession</u>

We just solved the <u>body frame</u> motion




direction CW vs. CCW

depends on $\lambda_3, \lambda_1$

$\vec{L}$ will also precess around some (different) axis.

( $\dot{\vec{L}} \neq 0$ in body frame )

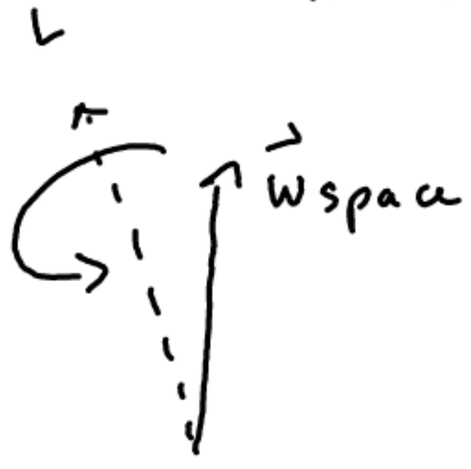
In the space frame:

$\vec{T} = 0 \longrightarrow \vec{L}$ const.

$T = \frac{1}{2}\vec{\omega} \cdot \vec{L}$ constant, too

$\downarrow$

$\vec{\omega}$ in $\vec{L}$ direction is const.




precesses around $\vec{L}$

new frequency