Centrifugal Force

$$\vec{F}_{CF} = m(\vec{\Omega}\times\vec{r})\times\vec{\Omega} \sim m r \Omega^2 \quad \text{order of mag}$$

$$\vec{F}_{cor} = m\dot{\vec{r}}\times\vec{\Omega} \sim m v \Omega \quad \text{order of mag}$$

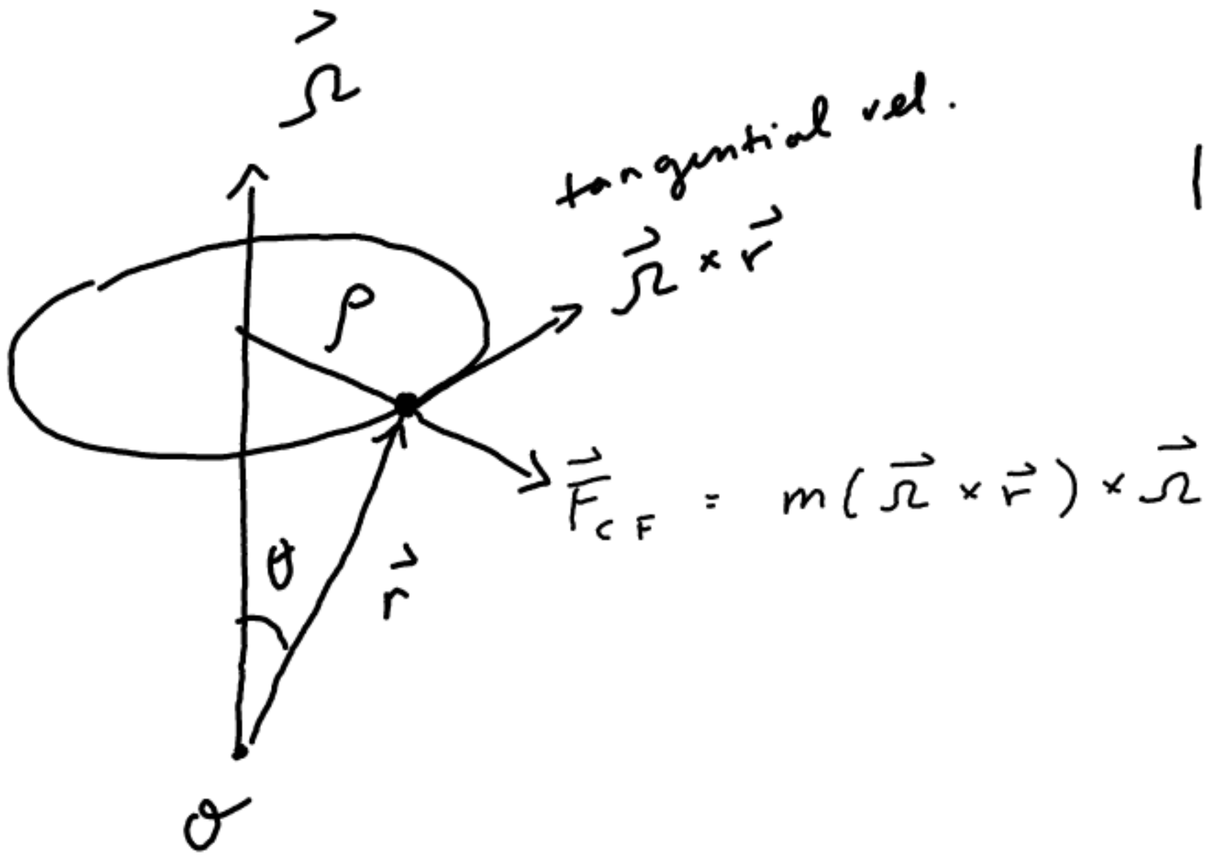
$$\frac{F_{cor}}{F_{CF}} \sim \frac{m v \Omega}{m r \Omega^2} = \frac{v}{r\Omega} = \frac{v}{V}$$

v in rotating frame

V of rotating frame

on Earth, $V \sim 1000$ mph

So we can ignore F_{cor} for anything with $v \ll 1000$ mph



VISUALIZE IT

$$|\vec{F}_{CF}| = m\Omega^2 r \sin\theta$$

$$= m\Omega^2 \rho$$

$$\vec{F}_{CF} = m\Omega^2 \rho\, \hat{\rho}$$

in cylindrical coords

Free-fall Acceleration

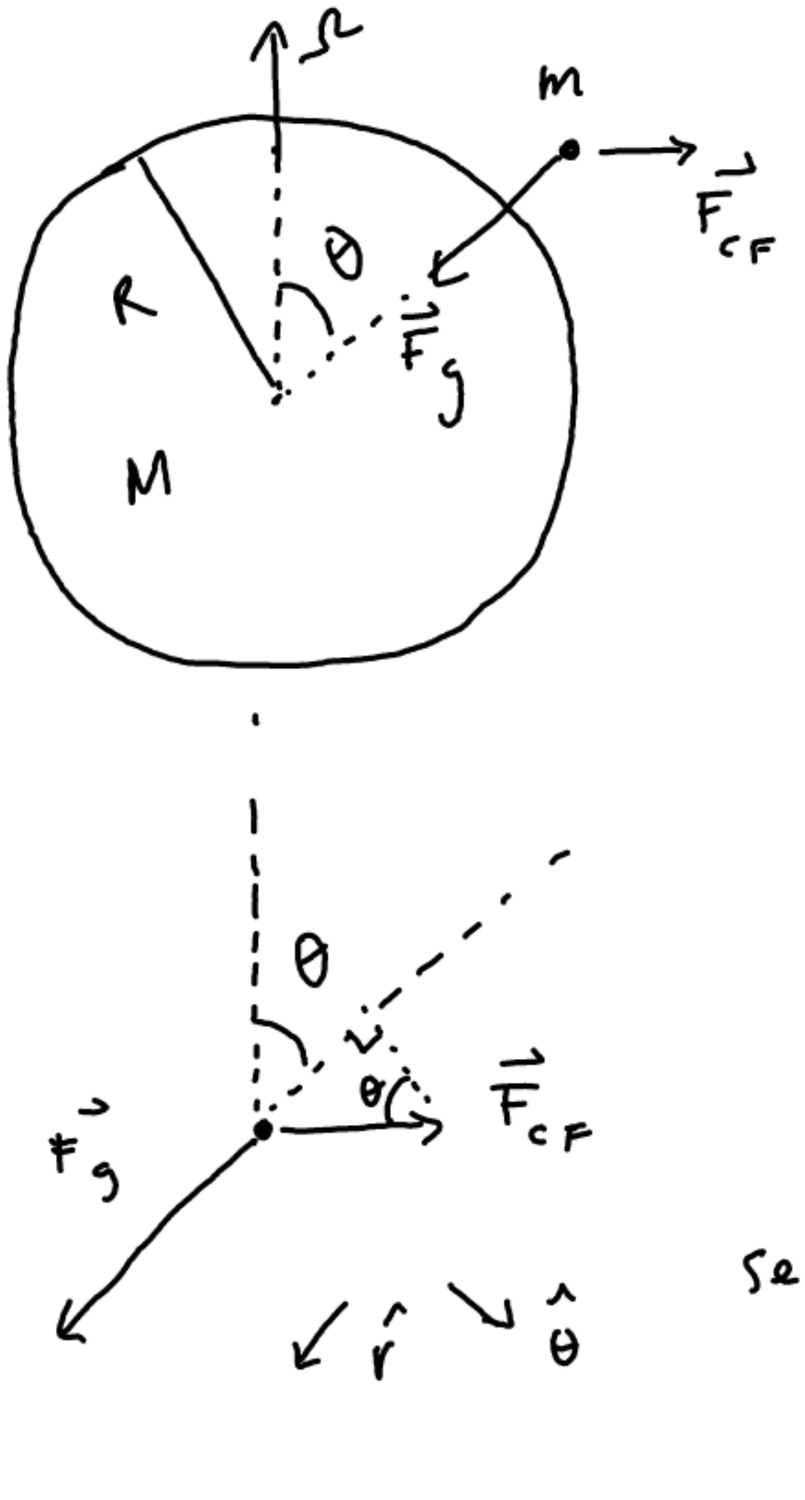
true free-fall acceleration is given by

$$m\ddot{\vec{r}} = \vec{F}_g + \vec{F}_{CF} \qquad \text{if } \vec{F}_{cor} \text{ small}$$

$$= m\vec{g}_o + \vec{F}_{CF}$$

$$\vec{g}_o = -\frac{GM}{R^2}\hat{r}$$

Free-body diagram



$$\vec{F}_{ell} = m\vec{g}_o + m\Omega^2 R\sin\theta\,\hat{\rho}$$

$$\vec{F}_{ell} = m\vec{g} \quad \text{where}$$

$$\vec{g} = \vec{g}_o + \Omega^2 R\sin\theta\,\hat{\rho}$$

separate $\vec{F}_{ell}$ into radial ($\hat{r}$) and tangential ($\hat{\theta}$) components:

$$F_r = g_o + \Omega^2 R\sin\theta\cdot\sin\theta$$

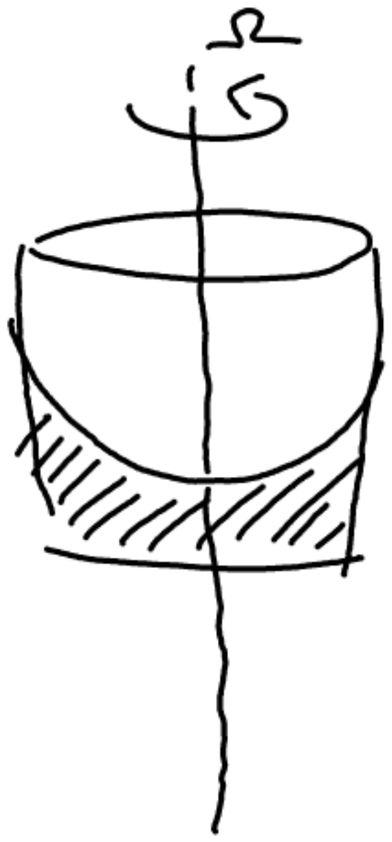
$$F_r = g_o + \Omega^2 R\sin^2\theta$$

min at poles
max at equator

$$F_\theta = \Omega^2 R\sin\theta\cos\theta$$

zero at poles + equator
max at 45°

Bucket of Water, spinning



Show that the surface of the water forms a parabola.

The surface at equilibrium will be an equipotential.

$$U_g = mgz$$

$$U_{CF} = -\int_0^\rho \vec{F}_{CF} \cdot d\vec{\rho} = -\int_0^\rho m\Omega^2 \rho \, d\rho = -\frac{1}{2} m\Omega^2 \rho^2$$

$$U = U_g + U_{CF}$$

$$= mgz - \frac{1}{2} m\Omega^2 \rho^2 = \text{const.}$$

$$z = \text{const.} + \frac{1}{2g}\Omega^2 \rho^2$$

is a parabola in ρ!