Homework 8: Rigid Rotors, Taylor Ch. 10

1. Moment of Inertia Tensor of a Uniform Solid Cone

   (a) Calculate the components of the moment of inertia tensor for an inverted uniform solid cone with mass M, height h, and radius R, with the coordinate system origin centered at its tip. Show that $I_{xx} = I_{yy} = \frac{3}{5}Mh^2 + \frac{3}{20}MR^2$, and $I_{zz} = \frac{3}{10}MR^2$.

   Hint 1: Write the volume element $dV = r\ dr\ d\phi\ dz$ in cylindrical coordinates r, ϕ, z, where $x = r\cos\phi$, $y = r\sin\phi$.

   Hint 2: Use the symmetry of the top to reduce the number of explicit integrations you must to, down to 2.

   (b) Is the moment of inertia tensor in principal component form?

2. Moment of Inertia Tensor for a Uniform Solid Cube

   (a) Calculate the components of the moment of inertia tensor for a uniform solid cube with mass M and length a, with the coordinate system origin about one corner. Show that $I_{xx} = I_{yy} = I_{zz} = \frac{2}{3}Ma^2$, and the remaining components $-\frac{1}{4}Ma^2$. Hint: you should only need to evaluate two integrals explicitly.

   (b) Solve the characteristic equation for the moment of inertia tensor of the cube in this coordinate system to find its eigenvalues.

   (c) Using the eigenvalues found in part b, solve for the principal axes of the cube in this coordinate system.

3. Symmetric top with non-zero torqe

   An axially symmetric space station (principle axis $\hat{e}_3$, and $\lambda_1 = \lambda_2$) is floating in free space. It has rockets mounted symmetrically on either side that are exerting a constant torque Γ about the symmetry axis. Solve Euler's equations exactly for $\vec{\omega}$ (relative to the body axis) and describe the motion. At $t = 0$, take $\vec{\omega} = (\omega_{1,0}, 0, \omega_{3,0})$.