1. Tutorial part III : adding friction

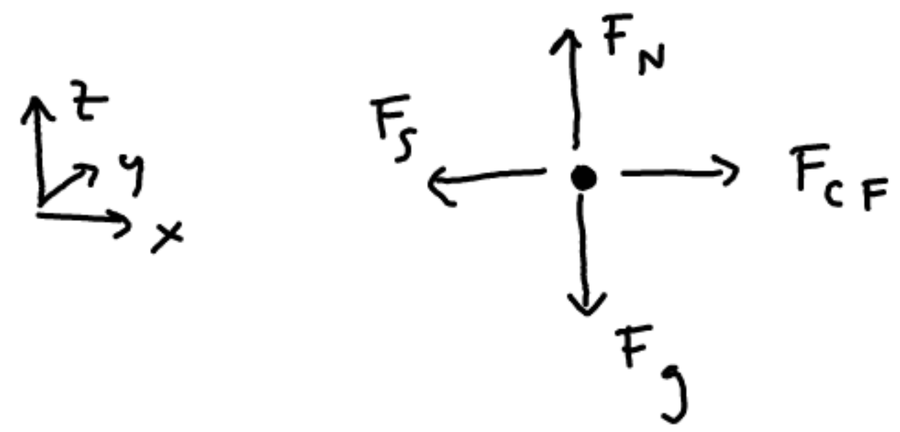


$\vec{\Omega} = -\Omega \hat{z}$

$g \otimes$

a) $v_0 = 0$, r_0

FBD in rotating frame



for the bug to <u>not slide</u> we must have

$$\Sigma \vec{F}_x = 0$$

$$\vec{F}_{CF} + \vec{F}_s = 0$$

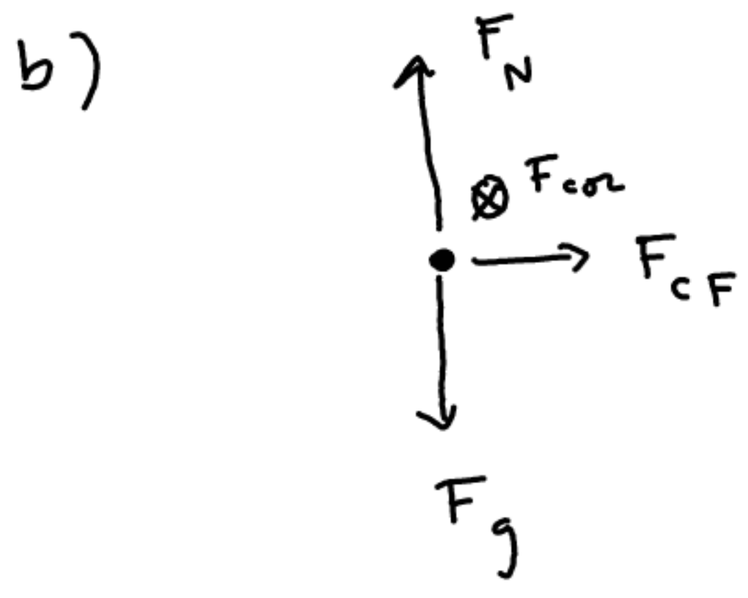
$$\left| m(\vec{\Omega} \times \vec{r}_0) \times \vec{\Omega} \right| - \mu_s m g = 0$$

$$m \Omega^2 r_0 = \mu_s m g$$

$$\boxed{r_0 = \mu_s g / \Omega^2}$$

max r for no slipping

b)

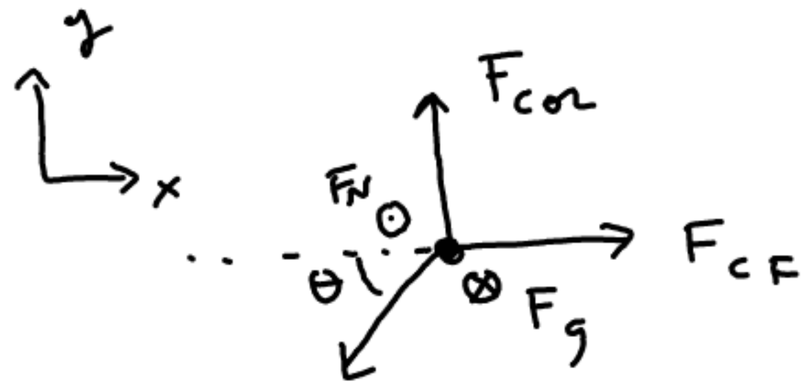


$\rightarrow v_0$ $\quad \downarrow \Omega$

$$\vec{F}_{cor} = m\vec{v}_0 \times \vec{\Omega}$$

$$= m v_0 \Omega \otimes$$

re-draw bird's eye view



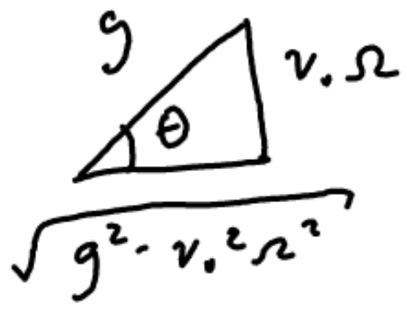
$$\Sigma F_x = F_{cF} - F_s \cos\theta = 0$$

$$= m\Omega^2 r - \mu_s mg \cos\theta = 0$$

$$\Sigma F_y = F_{cor} - F_s \sin\theta = 0$$

$$mv_0\Omega - \mu_s mg \sin\theta = 0$$

$$\sin\theta = \frac{v_0\Omega}{g}$$

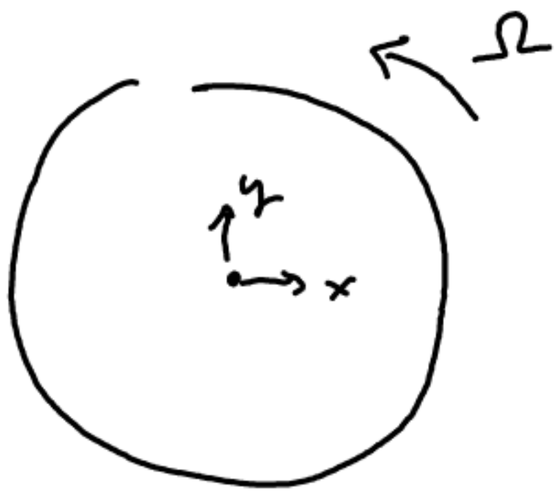


$$r = \frac{1}{\Omega^2}\mu_s g \cos\left(\sin^{-1}\left(\frac{v_0\Omega}{g}\right)\right)$$

$$= \frac{1}{\Omega^2}\mu_s g \cdot \frac{\sqrt{g^2 - v_0^2\Omega^2}}{g}$$

$$r = \frac{\mu_s}{\Omega^2}\sqrt{g^2 - v_0^2\Omega^2}$$

2.



a) $m\ddot{\vec{r}} = \cancel{\vec{F}}^{\,0} + 2m\dot{\vec{r}} \times \vec{\Omega} + m(\vec{\Omega} \times \vec{r}) \times \vec{\Omega}$

$\vec{r} = (r\cos\theta,\ r\sin\theta) = (x, y)$

$\hookrightarrow \quad \vec{\Omega} \times \vec{r} = \Omega\hat{z} \times (x, y)$

$= \Omega(\hat{z} \times \vec{x} + \hat{z} \times \vec{y})$

$= \Omega(x\hat{y} - y\hat{x})$

$(\vec{\Omega} \times \vec{r}) \times \vec{\Omega} = \Omega(-y, x) \times \Omega\hat{z}$

$= \Omega^2(-y\hat{x} \times \hat{z} + x\hat{y} \times \hat{z})$

$= \Omega^2(-y(-\hat{y}) + x(\hat{x}))$

$= \Omega^2(x, y) = \Omega^2\vec{r}$

$\dot{\vec{r}} = (\dot{x}, \dot{y}) \rightarrow \quad \dot{\vec{r}} \times \vec{\Omega} = \dot{x}(\hat{x} \times \Omega\hat{z}) + \dot{y}(\hat{y} \times \Omega\hat{z})$

$= \Omega\dot{x}(-\hat{y}) + \Omega\dot{y}(\hat{x})$

$= \Omega(\dot{y}, -\dot{x})$

$$m\ddot{x} = 2m\Omega\dot{y} + m\Omega^2 x$$

$$m\ddot{y} = -2m\Omega\dot{x} + m\Omega^2 y$$

b) let $\eta = x + iy$

multiply the y-eqn by i and add the x, y-eqns:

$$m\ddot{x} + im\ddot{y} = 2m\Omega\dot{y} - i\,2m\Omega\dot{x} + m\Omega^2 x + im\Omega^2 y$$

$$\ddot{x} + i\ddot{y} = 2\Omega i(-i\dot{y} - \dot{x}) + \Omega^2(x + iy)$$

$$\ddot{\eta} = -2\Omega i\dot{\eta} + \Omega^2\eta$$

$$\ddot{\eta} + 2\Omega i\dot{\eta} - \Omega^2\eta = 0$$

non-homogeneous diff. eqn.

let $\eta(t) = Ae^{-i\alpha t}$

$$-\alpha^2 + 2\Omega i(-i\alpha) - \Omega^2 = 0$$

$$-\alpha^2 + 2\Omega\alpha - \Omega^2 = 0$$

$$(\alpha - \Omega)^2 = 0$$

$$\alpha = \Omega$$

$\eta(t) = Ae^{-i\Omega t}$ is one solution

second solution

$$\eta(t) = Bt\, e^{-i\alpha t}$$

$\cos\Omega t - i\sin\Omega t$

$-\cos\Omega t + i\sin\Omega t$

total solution:

$$\eta(t) = Ae^{-i\Omega t} + Bt\, e^{-i\Omega t}$$

$$= e^{-i\Omega t}(A + Bt)$$

c) at $t = 0$, $\eta = x_0$

$$\dot{\eta} = v_{ox} + iv_{oy}$$

$$\eta(0) = A = x_0$$

$$\dot{\eta} = -i\Omega e^{-i\Omega t}(A + Bt) + Be^{-i\Omega t}$$

$$\dot{\eta}(0) = -i\Omega A + B = v_{ox} + iv_{oy}$$

$$-i\Omega x_0 + B = v_{ox} + iv_{oy}$$

$$B = v_{ox} + i(v_{oy} + \Omega x_0)$$

$$\eta(t) = e^{-i\Omega t}\left(x_0 + v_{ox}t + i(v_{oy} + \Omega x_0)\, t\right)$$

$$\eta(t) = e^{-i\Omega t}\left(x_0 + v_{0x} t + i(v_{0y} + \Omega x_0)\, t \right)$$

$$x(t) = \mathrm{Re}(\eta)$$

$$= \mathrm{Re}\left(\cos\Omega t - i \sin\Omega t\right)\left(x_0 + v_{0x} t + i\,(v_{0y} + \Omega x_0)\, t\right)$$

$$= x_0 \cos\Omega t + v_{0x} t \cos\Omega t + (v_{0y} + \Omega x_0)\, t \sin\Omega t$$

$$x(t) = (x_0 + v_{0x} t)\cos\Omega t + (v_{0y} + \Omega x_0)\, t \sin\Omega t \quad \checkmark$$

$$y(t) = \mathrm{Im}(\eta)$$

$$= (v_{0y} + \Omega x_0)\, t \cos\Omega t - x_0 \sin\Omega t - v_{0x} t \sin\Omega t$$

$$= -(x_0 - v_{0x} t)\sin\Omega t + (v_{0y} + \Omega x_0)\, t \cos\Omega t \quad \checkmark$$

d) For $t \to \infty$, ignore terms not multiplied by t.

$$x(t) = v_{0x} t \cos\Omega t + (v_{0y} + \Omega x_0)\, t \sin\Omega t$$

$$y(t) = v_{0x} t \sin\Omega t + (v_{0y} + \Omega x_0)\, t \cos\Omega t$$

radius is linear in t

spiraling outward

3. Magnitude of Coriolis + Centrifugal Forces



Find F_{cor}, F_{CF} right before the ball hits the ground.

$$F_{cor} \sim 2mv\Omega = 2m\sqrt{2g\Delta h}\,\Omega$$

$$\sqrt{2g(60m)} \sim 34\,m/s$$

$$\Omega = \frac{2\pi}{1\,day} \sim 7.3\times10^{-5}\,s^{-1}$$

$$\downarrow$$

$$a_{cor} \sim 2\sqrt{2g\Delta h}\,\Omega$$

$$= 2\cdot(34\,m/s)(7.3\times10^{-5}/s)$$

$$\frac{a_{cor}}{g} \sim 5\times10^{-4} \quad \text{pretty small}$$

$$F_{CF} \sim m\Omega^2 r = m\Omega^2(R_E)$$

$$\frac{a_{CF}}{g} = \frac{\Omega^2 R_E}{g} = \frac{(7.3\times10^{-5}/s)(6.371\times10^{6}\,m)}{9.8\,m/s^2}$$

$$= 3\times10^{-3} \quad \text{small, but less small}$$

important if : no g (terminal velocity, wind)

large v

3. Coriolis Deflection in Free Fall

a) $v_r = -gt$ no θ or φ velocity (no Coriolis effect)

b) local spherical $\longrightarrow$ global cartesian

$$\begin{pmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\varphi & \cos\theta\cos\varphi & -\sin\varphi \\ \sin\theta\sin\varphi & \cos\theta\sin\varphi & \cos\varphi \\ \cos\theta & -\sin\theta & 0 \end{pmatrix} \begin{pmatrix} \hat{r} \\ \hat{\theta} \\ \hat{\varphi} \end{pmatrix}$$

$$\hat{z} = \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}$$

if $\vec{\omega} = \omega\hat{z}$ then $\vec{\omega} = \omega(\cos\theta\,\hat{r} - \sin\theta\,\hat{\theta})$

c) If $\lambda = \frac{\pi}{2} - \theta$ then

$$\cos\theta = \cos(\lambda - \pi/2) = \sin\lambda$$

$$\sin\theta = \sin(\lambda - \pi/2) = -\cos\lambda$$

$$\vec{\omega} = \omega(\sin\lambda\,\hat{r} + \cos\lambda\,\hat{\theta})$$

d) $\vec{a}_c(t) = -2\vec{\omega}\times\vec{v}$

$$= -2\omega(\sin\lambda\,\hat{r} + \cos\lambda\,\hat{\theta}) \times (-gt\,\hat{r})$$

$$= 0 + 2\omega g t\cos\lambda\,(+\hat{\varphi})$$

$$\vec{a}_c(t) = +2g\omega t\cos\lambda\,\hat{\varphi}$$

$$\vec{a}_c(t) = -2g\omega t \cos\lambda \; \hat{\varphi}$$

integrating twice in t gives

$$\Delta x(\lambda) = \frac{-1}{3} g\omega t_f^3 \cos\lambda$$

$$t_f = \sqrt{\frac{2h}{g}}$$

e) $\Delta x = \frac{1}{3}(9.8\,m/s^2)(7.3\times10^{-5}\,rad/s)\left(\frac{2\times60m}{9.8\,m/s^2}\right)^{3/2}\cos(41.6°)$

$= 0.763$ cm $+\hat{\varphi}$ dir

↑ east

This is very small and hard to measure w/in a reasonable uncertainty.

If $h = 828$ m, $\lambda = 25.2°$

↳ $\Delta x = 52.2$ cm east

might be able to measure this!