Homework 2: Lagrangian Mechanics, Taylor Ch. 7

1. Write down the Lagrangian for a one-dimensional particle moving along the x-axis and subject to a force $F = -kx$ (with k positive). Find the Lagrange equation of motion and solve it.

2. Two masses on a spring

   (a) Write down the Lagrangian $\mathcal{L}(x_1, x_2, \dot{x_1}, \dot{x_2})$ for two particles of equal masses, $m_1 = m_2 = m$, confined to the x-axis and connected by a spring with potential energy $U = kx^2$. [Here x is the extension of the spring, $x = (x_1 - x_2 - l)$, where l is the spring's unstretched length, and I assume that mass 1 remains to the right of mass 2 at all times.]

   (b) Rewrite $\mathcal{L}$ in terms of the new variables $X = \frac{1}{2}(x_1 + x_2)$ (the CM position) and x (the extension given in part a), and write down the two Lagrange equations for X and x .

   (c) Solve for $X(t)$ and $x(t)$ and describe the motion.

3. Write down the Lagrangian for a cylinder (mass m , radius R, and moment of inertia I) that rolls without slipping straight down an inclined plane which is at an angle α from the horizontal. Use as your generalized coordinate the cylinder's distance x measured down the plane from its starting point. Write down the Lagrange equation and solve it for the cylinder's acceleration $\ddot{x}$.

4. Consider a double Atwood machine constructed as follows: A mass $4m$ is suspended from a string that passes over a massless pulley on frictionless bearings. The other end of this string supports a second similar pulley, over which passes a second string supporting a mass of $3m$ at one end and m at the other. Using two suitable generalized coordinates, set up the Lagrangian and use the Lagrange equations to find the acceleration of the mass $4m$ when the system is released. Explain why the top pulley rotates even though it carries equal weights on each side.