Calculating Electric Field

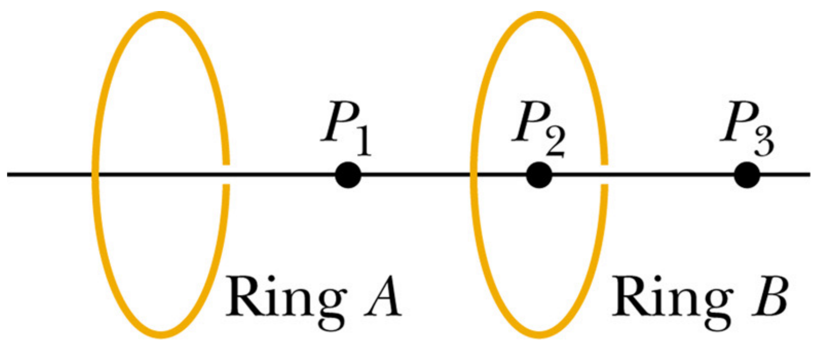
PHY 114-01

Fall 2026

Warm up problem!

Rings A and B carry uniform linear charge density. Rank the points according to the magnitude of electric field, greatest first.

1. ***P_1, P_2, P_3***
2. ***P_2, P_3, P_1***
3. ***P_3, P_1, P_2***
4. ***$P_1 = P_2 = P_3$***
5. ***P_3, P_2, P_1*** ✓

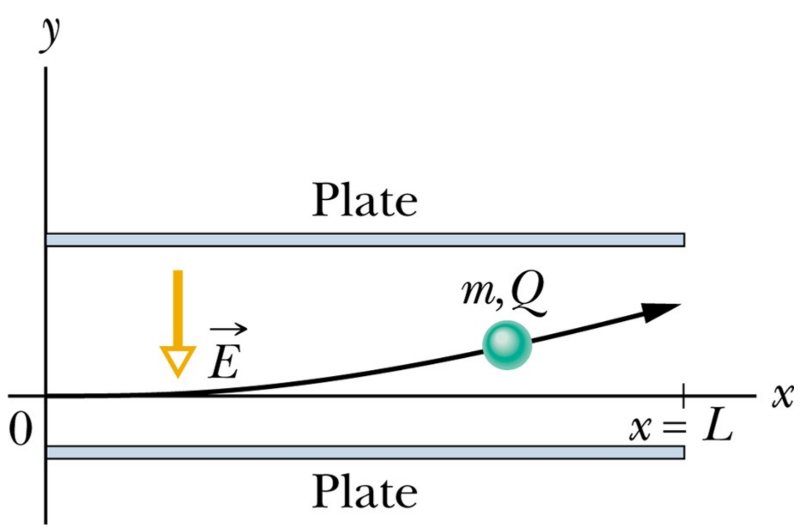


Agenda

- Last time:
  - Ring of charge
  - Charged particles moving in fields
- This time:
  - Infinite plane of charge   Needed for homework
  - Sphere of charge
- Recitation:
  - Charged particles moving in fields   Needed for homework

Charged particle in a uniform field

- Constant field = constant force = constant acceleration
- What tool from mechanics do we have to solve this problem?



Go to recitation for a worked example!

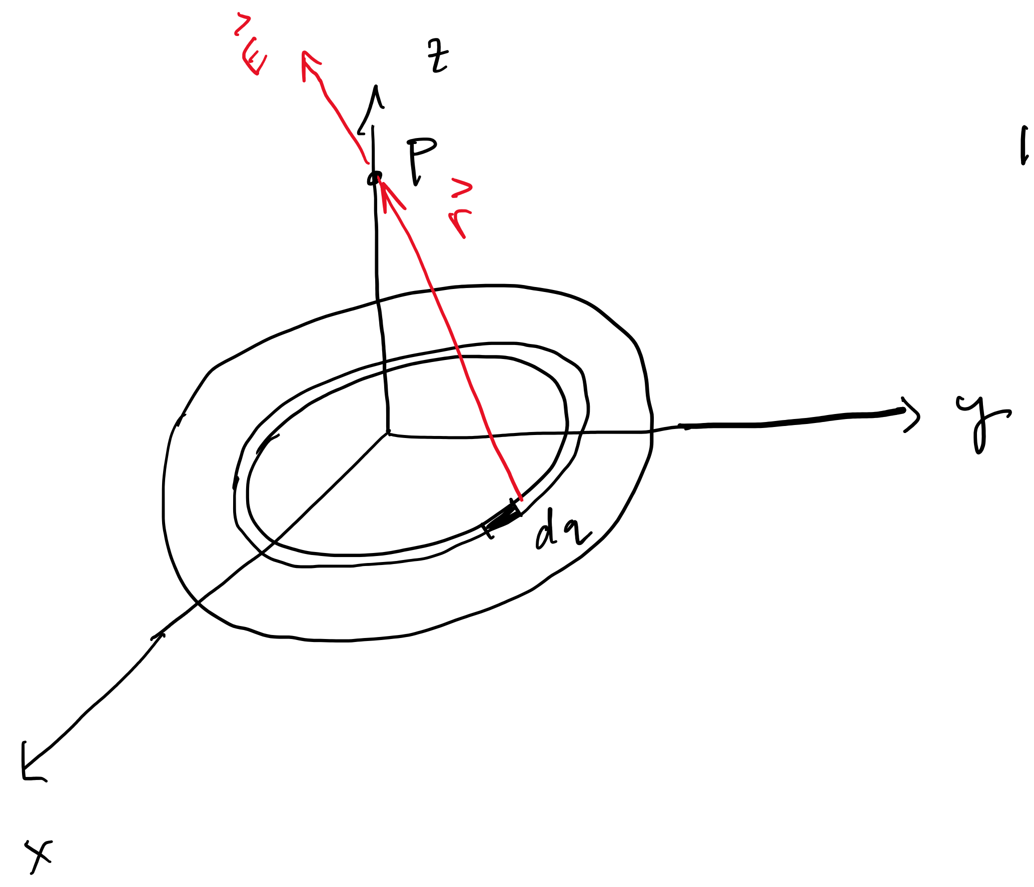
Example: infinite plane of charge

Electric field from uniform infinite plane

Sunday, September 20, 2026   5:03 PM

Integrating the infinite ring of charge wasn't too bad.

Let's extend the concept to make a disk of charge, and then take $R \to \infty$ to make an infinite plane.



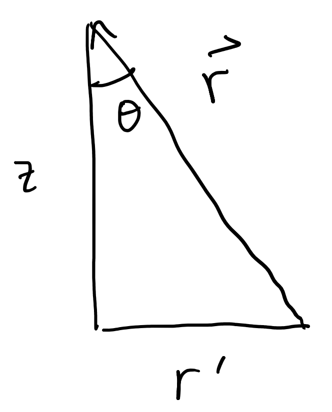
1. dq is now the same that we had before, but multiplied by some small change in radius (r')

$dq = \sigma dA$ $dA = 2\pi r' dr'$ ← tiny change in r'

↑ circumference of circle r'

$$dq = \sigma \cdot 2\pi r' dr'$$

2. $$d\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{dq}{r^2} \hat{r} = \frac{1}{4\pi\varepsilon_0} \frac{dq}{r^3} \vec{r}$$



I am only concerned with the z-component because the radial (xy) components will all cancel.

$$dE_z = \frac{1}{4\pi\varepsilon_0} \frac{dq}{r^3} \vec{z} = \frac{1}{4\pi\varepsilon_0} \frac{\sigma \cdot 2\pi r' dr'}{(r'^2 + z^2)^{3/2}} z$$

3. Integrate

$$E_z = \int_0^R \frac{1}{4\pi\varepsilon_0} \frac{2\pi\sigma r' z}{(r'^2 + z^2)^{3/2}} dr'$$

let $u = r'^2 + z^2$

$du = 2r' dr$

$$E_z = \int_{z^2}^{R^2+z^2} \frac{\sigma z}{4\varepsilon_0} \frac{du}{u^{3/2}} = \frac{\sigma z}{4\varepsilon_0} \left[u^{-1/2} \cdot (-2) \right]_{z^2}^{R^2+z^2}$$

$$= -\frac{\sigma z}{2\varepsilon_0} \left[\frac{1}{\sqrt{R^2+z^2}} - \frac{1}{\sqrt{z^2}} \right]$$

let $R \to \infty$ $-\frac{\sigma z}{2\varepsilon_0}\left(0 - \frac{1}{z}\right) = \frac{\sigma}{2\varepsilon_0}$

$$\boxed{\vec{E} = \frac{\sigma}{2\varepsilon_0} \hat{z}}$$

Notice that this result is constant!