Calculating the Electric Field

PHY 114-01

Fall 2026

Welcome!

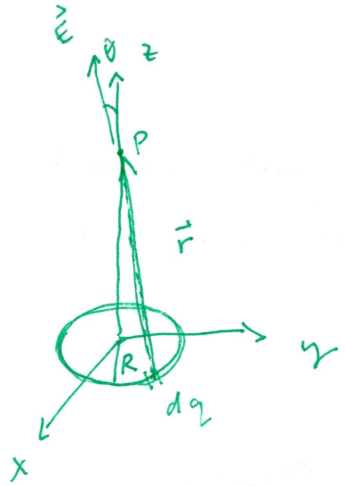
- Take a handout (put your name on it)
- Sit next to at least one person to work with

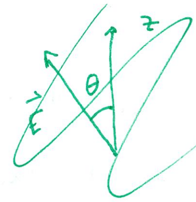
Agenda

- Last time:
  - Defining continuous charge distributions
  - Steps for integrating
  - Finite line example

- This time:
  - Ring of charge example
  - Plane of charge example
  - Particle moving in an electric field

Example: uniform ring of charge

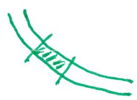




$\cancel{E_r = E \sin\theta}$

$\cancel{E_z = E \cos\theta}$

1. Find dq:



small segment of a ring

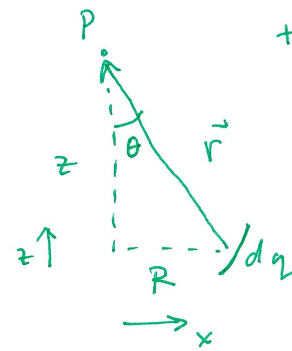
$$ds = R\,d\theta$$

$$dq = \lambda R\,d\theta$$

$$= \frac{Q}{2\pi R} R\,d\theta$$

$$dq = \frac{Q}{2\pi}\,d\theta$$

2. Find $\vec{E}_{dq}$: ~~See diagram above~~



take a slice along the x-axis.

$$|\vec{r}| = \sqrt{R^2 + z^2}$$

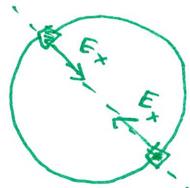
$$\vec{r} = \sqrt{R^2+z^2}\,(-R\hat{x} + z\hat{z})$$

$$\vec{E}_{dq} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^3}\vec{r}$$

$$\vec{E}_{dq} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{2\pi}\,d\theta\right)\frac{1}{(R^2+z^2)^{3/2}}(-R\hat{x} + z\hat{z})$$

3. Integrate over all dq:

First consider symmetry. The "x" components around the circle will all cancel each other so we can ignore that term.

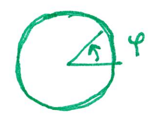


cancel!

We are left with

φ is the angle around the circle

$$E_z = \frac{Q}{8\pi^2\varepsilon_0}\int_0^{2\pi}\frac{z}{(R^2+z^2)^{3/2}}\,d\varphi$$



$$= \frac{Q}{8\pi^2\varepsilon_0}\cdot 2\pi\,\frac{z}{(R^2+z^2)^{3/2}}$$

$$\boxed{E_z = \frac{Q}{4\pi\varepsilon_0}\,\frac{z}{(R^2+z^2)^{3/2}}}$$

for $z \gg R$, we get

$$E_z = \frac{Q}{4\pi\varepsilon_0}\,\frac{1}{z^2}$$

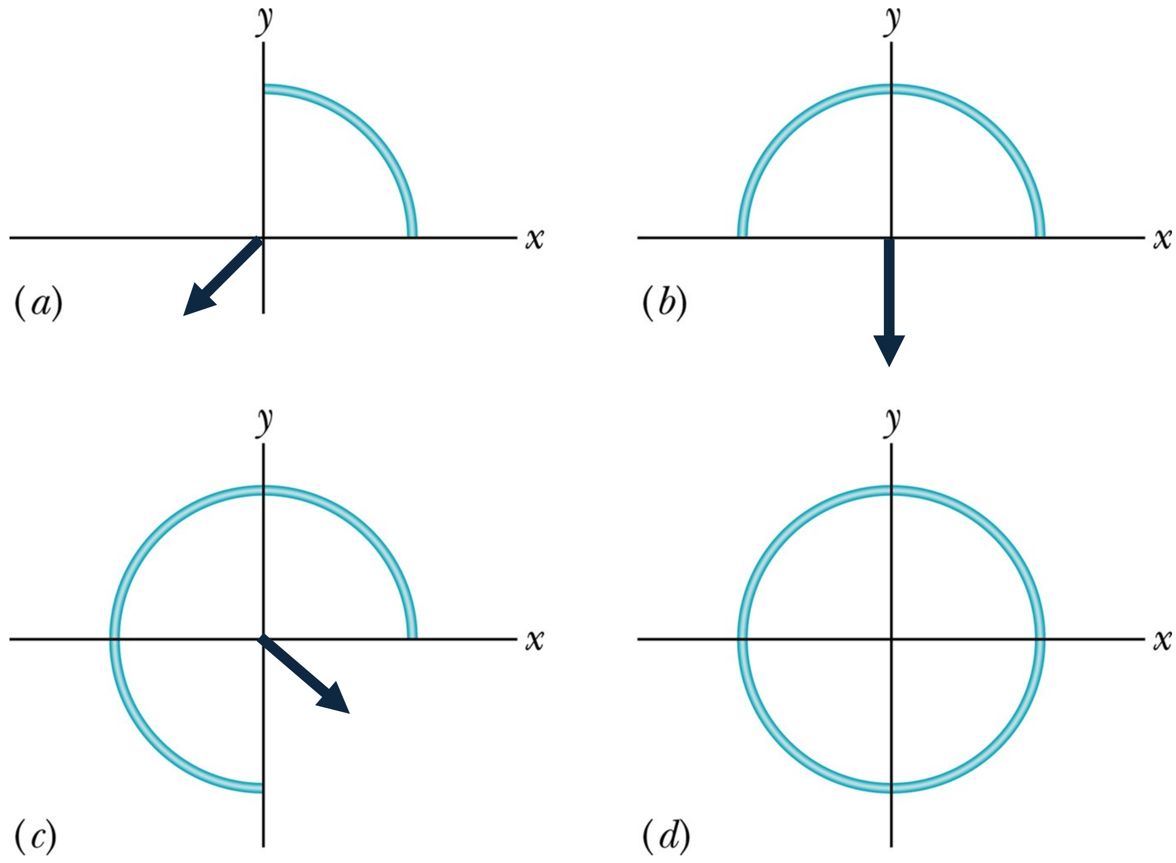
which looks just like a point charge!

Some variations for you to think about

- Disk of charge
- Annular disk of charge
- Nonuniform charge distribution (varying in radius or angle)
- Off-center field calculation

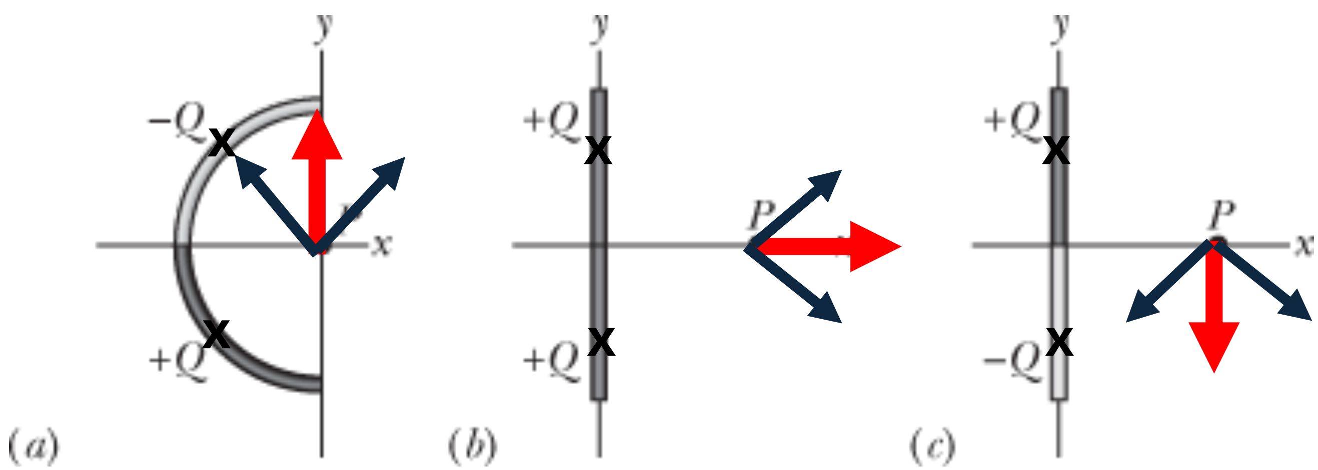
Exercise Electric Field

Circular arcs carry uniform, positive linear charge density.
Draw the direction of the electric field at the center (origin).



Checkpoint 2: E-Fields of line charges

The figure here shows three nonconducting rods, one circular and two straight. Each has a uniform charge of magnitude *Q* along its top half and another along its bottom half. For each rod, what is the direction of the net electric field at point *P*?

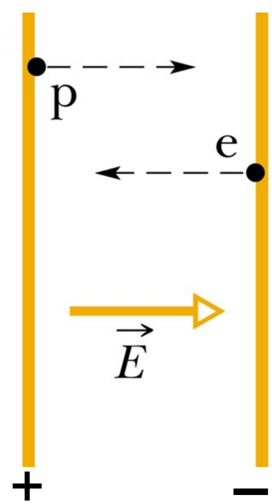


Motion in Electric Fields

Electric fields exert a force on a charge as

$$\vec{E} = \vec{F}/q, \quad \vec{F} = q\vec{E}$$

The motion will be governed by Newton's laws

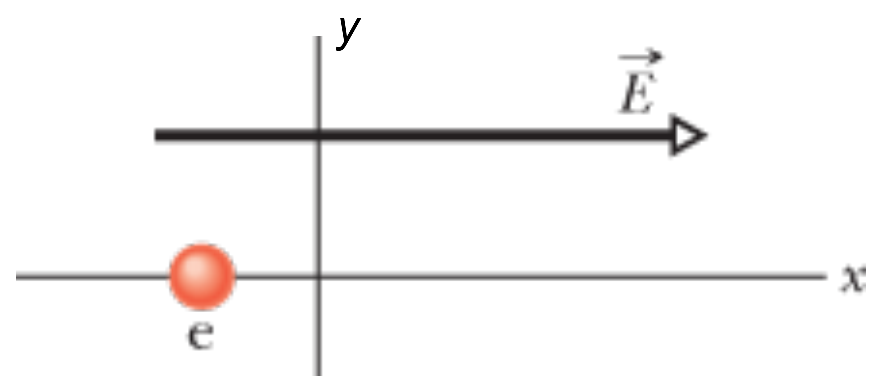


+/- plates

☑ *Checkpoint* 3: Motion in E-Fields

In the figure, what is the direction of the electrostatic force on the electron due to the external electric field shown?

$$\vec{F} = q\,\vec{E}$$

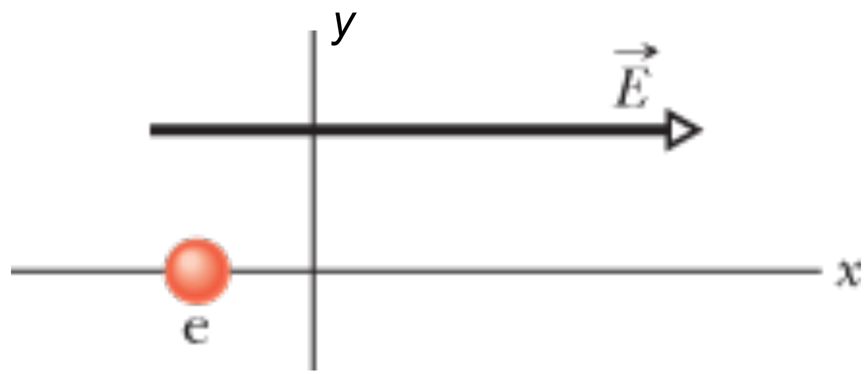


A. Up
B. Down
C. Left
D. Right
E. None of the above

☑*Checkpoint* 3: Motion in E-Fields

In which direction will the electron accelerate if it is moving parallel to the *y* axis before it encounters the external field?

$$\vec{F} = q\,\vec{E}$$

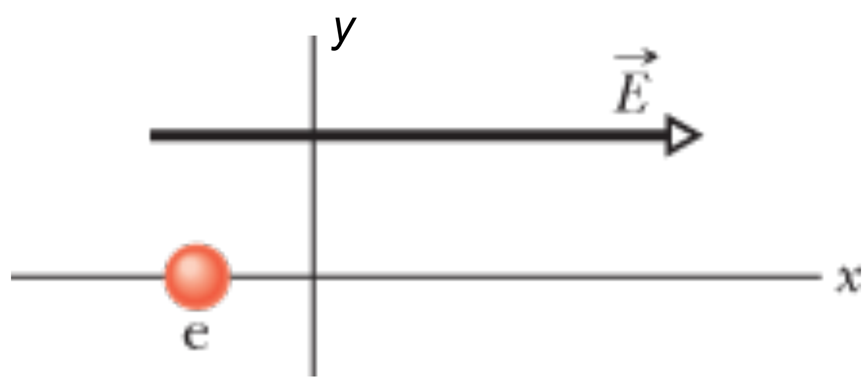


A. Up
B. Down
C. Left
D. Right
E. None of the above

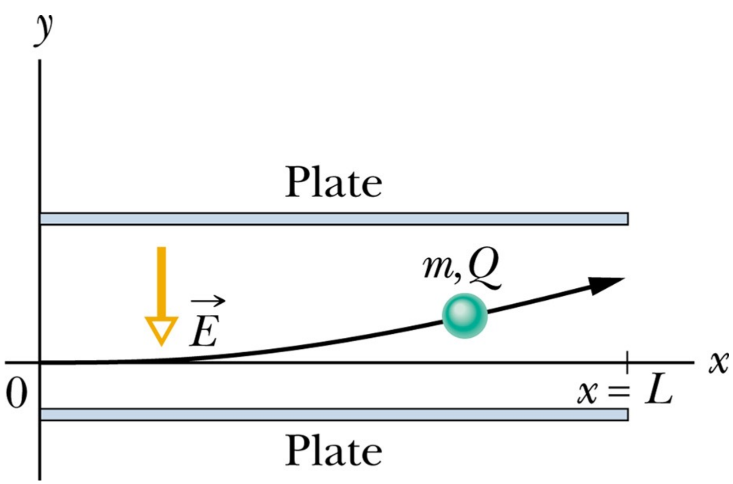
Checkpoint 3: Motion in E-Fields

If, instead, the electron is initially moving rightward, will its speed increase, decrease, or remain constant?

$$\vec{F} = q\vec{E}$$



A. Increase forever
B. Decrease to constant speed
C. Decrease to stop
D. Decrease to stop, then turn around and increase
E. None of the above



y
Plate
m, Q
E
x
0
x = L
Plate