Calculating the Electric Field

PHY 114-01

Fall 2026

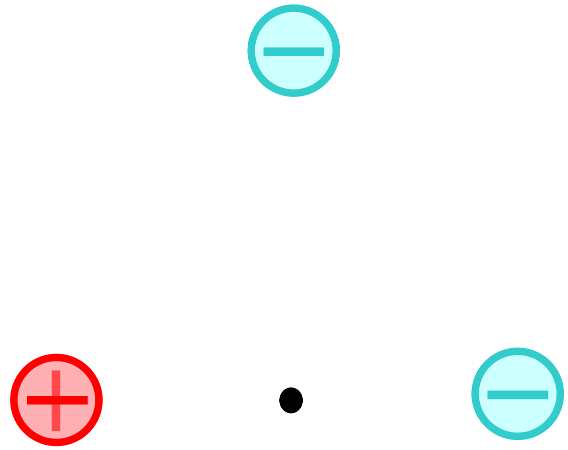
Welcome!

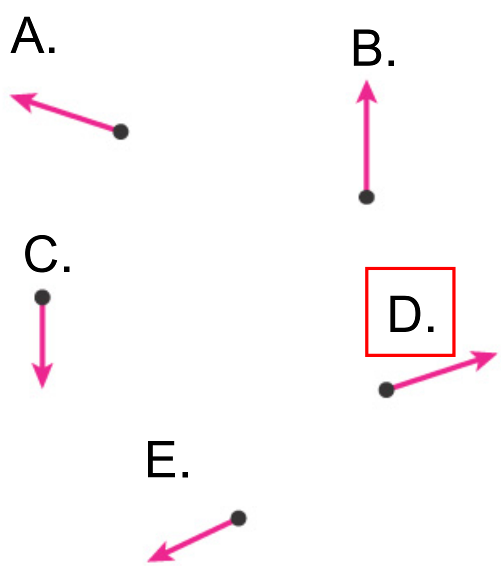
- Take a handout (put your name on it)
- Sit next to at least one person to work with

Agenda

- Last time:
  - Electric field definition
- This time:
  - Electric field calculations from continuous charge distributions

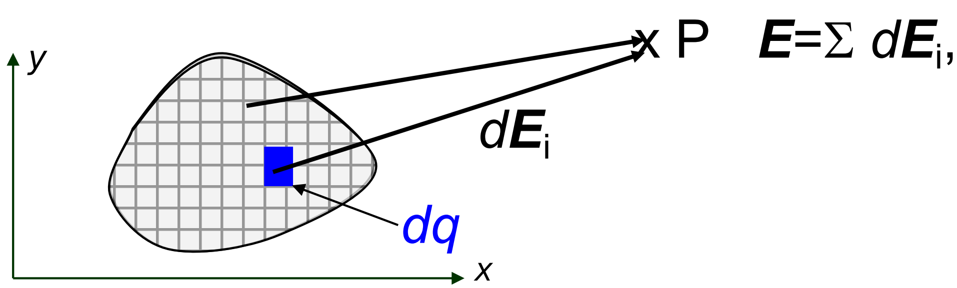
Now, Tom changes one of the positive charges on the bottom to negative, as shown below. At the position of the dot, the ***electric field*** points approximately





Charge distribution

- Any configuration of many charges with a fixed spatial relationship
- Usually, in the real world, continuous



Electric Field Due to Many Charges

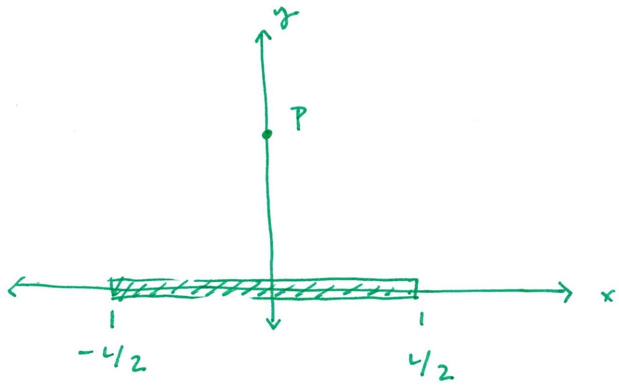
- **When dealing with a *continuous charge distribution* it is convenient to talk about the charge on an object in terms of a *charge density* rather than the total charge.**
  - **imagine a line with total charge q and length l.**
    - **define *linear charge density* $\lambda = q/l$.**

Some Measures of Electric Charge

NAME	symbol	SI unit
total charge	q	C
linear charge density	λ	C/m
surface charge density	σ	C/m^2
volume charge density	ρ	C/m^3

Example: uniform line of charge

Electric Field from a Line of Charge



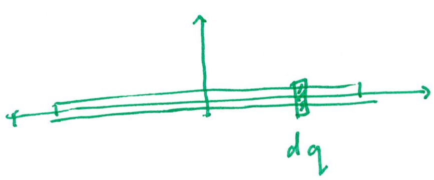
Find the electric field at P.

The line of charge has length L and total charge Q uniformly distributed.

Steps:

1. Find tiny chunk of charge dq.
2. Write down electric field from point charge dq.
3. Integrate over all dq.

1. Find dq.

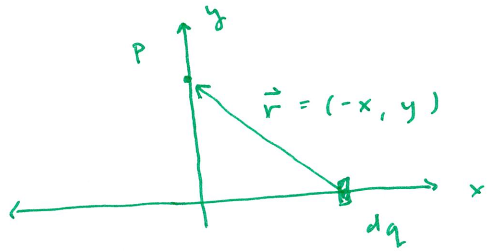


charge density $\lambda = Q/L$

$dq = \lambda \, dx$

$[C] = \frac{[C]}{[m]}[m]$ units work!

2. Write down electric field from dq.



dq is at some position x

P is at some position y

$\vec{r} = (-x, y)$

$$\vec{E}_{dq}(P) = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^3}\vec{r}$$

$$\vec{E}_{dq}(P) = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{(x^2+y^2)^{3/2}}(-x, y)$$

3. Integrate over all dq.

* special note: this integral is tricky, and you won't be asked to do anything this complicated on an exam. You might have to take a simpler integral though, so pay attention to the method.

Need to integrate two components to get E_x and E_y.

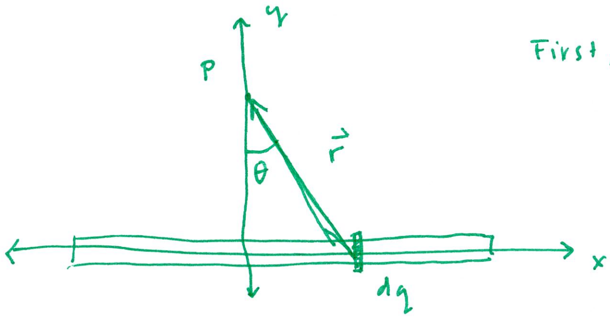
We can skip E_x because of symmetry. Any dq located at $(-x)$ will cancel the field from a dq at $(+x)$.

So, $E_x = 0$.

Then we "just" have to integrate for E_y.

We will skip most of this calculation in lecture, but here it is for your reference:

$$E_y = \int_{-L/2}^{L/2} \frac{1}{4\pi\varepsilon_0} \frac{\lambda \, dx}{(x^2 + y^2)^{3/2}} \cdot y$$



First, make a trig substitution:

$$\tan\theta = x/y$$

$$\sec^2 d\theta = \frac{dx}{y} \quad \leftarrow \frac{d}{d\theta}\tan\theta = \sec^2\theta$$

$$dx = y \sec^2\theta$$

$$E_y = \frac{\lambda}{4\pi\varepsilon_0} \int_{x_1 = -L/2}^{x_2 = L/2} \frac{y^2 \sec^2\theta \, d\theta}{(y^2 \tan^2\theta + y^2)^{3/2}}$$

Now this is an integral in θ, but eventually we will convert back to x. Don't worry about the integration bounds right now.

$$= \frac{\lambda}{4\pi\varepsilon_0} \int_1^2 \frac{y^2 \sec^2\theta \, d\theta}{y^3 (\tan^2\theta + 1)^{3/2}} \quad \leftarrow \tan^2\theta + 1 = \sec^2\theta$$

$$= \frac{\lambda}{4\pi\varepsilon_0} \int_1^2 \frac{1}{y} \frac{\sec^2\theta}{\sec^3\theta} d\theta = \frac{\lambda}{4\pi\varepsilon_0} \int_1^2 \frac{1}{y} \frac{1}{\sec\theta} d\theta$$

$$= \frac{\lambda}{4\pi\varepsilon_0} \int_1^2 \frac{1}{y} \cos\theta \, d\theta = \frac{\lambda}{4\pi\varepsilon_0} \frac{1}{y} \sin\theta \Big|_1^2$$

now convert back to x.

$$E_y = \frac{\lambda}{4\pi\varepsilon_0} \frac{1}{y} \left[\frac{x}{\sqrt{x^2 + y^2}} \right]_{-L/2}^{L/2} \rightarrow E_y = \frac{\lambda}{4\pi\varepsilon_0} \frac{1}{y} \left[\frac{L/2}{\sqrt{L^2/4 + y^2}} - \frac{(-L/2)}{\sqrt{L^2/4 + y^2}} \right]$$

If I ask you about a line of charge on an exam, I will give you this formula.

$$\boxed{E_y = \frac{\lambda}{4\pi\varepsilon_0} \frac{1}{y} \frac{L}{\sqrt{L^2/4 + y^2}}}$$

Some variations for you to think about

- Infinite line of charge
- Non-uniform line of charge (charge density depends on x)
- Calculate the field off-center or at the pole of the line/rod

Name: ____

1. (0.5pt completion) Write down the charge density for the following uniform charge distributions:
   (a) A line of charge
   $\lambda = Q/L$
   (b) A plane of charge
   $\sigma = Q/A$
   (c) A solid sphere of charge
   $\rho = Q/V = Q/(4/3\pi R^3)$
   (d) A hollow sphere of charge
   $\sigma = Q/A = Q/(4\pi R^2)$

2. (0.5pt completion) Write down dq for the same charge distributions above.
   (a) A line of charge
   $dq = \lambda dx = Q/Ldx$
   (b) A plane of charge
   $dq = \sigma dA = Q/Adxdy$
   (c) A solid sphere of charge
   $dq = \rho dV = Q/(4/3\pi R^3)r^2 \sin\phi dr d\phi d\theta$
   Here I have used the volume element of a sphere.
   (d) A hollow sphere of charge
   $dq = \sigma dA = Q/(4\pi R^2)R^2 \sin\phi d\phi d\theta$
   Here I have used the surface element of a sphere.

3. (1pt correctness) Write down the three steps for integrating a charge distribution.

   1. Write down the charge element dq.
   2. Write down the equation for the charge element contribution to the field dE, using the equation for the field of a point charge.
   3. Integrate over all the dq to get the total field.