Using Coulomb’s Law

PHY 114-01

Fall 2026

Welcome!

- Take a handout (put your name on it)
- Sit next to at least one person to work with

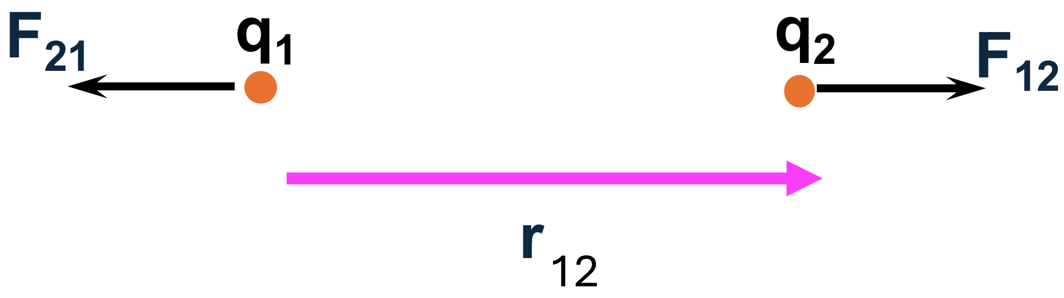
- Discuss: How would I find the force on a charge due to the presence of two other charges?

Agenda

- Last time:
  - The electric force law
  - Distance and charge scaling
  - Vector review
- This time:
  - Solve some problems!

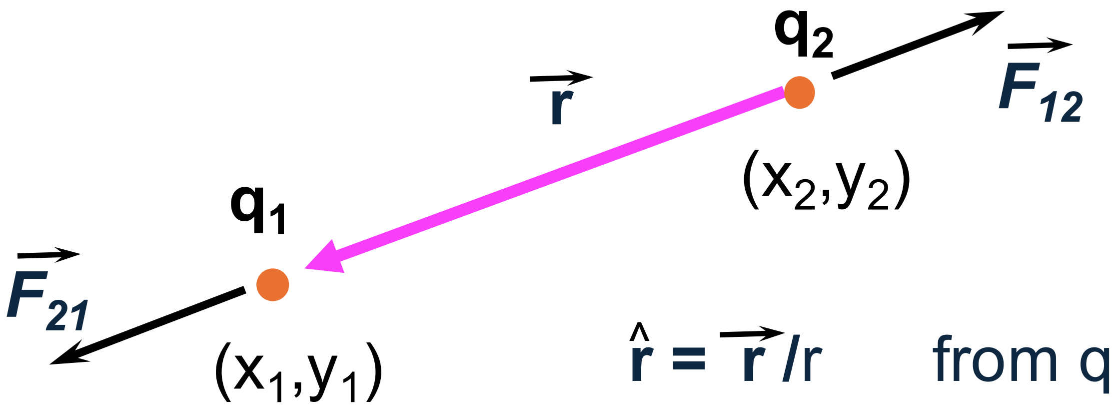
Coulomb's Law

- Congratulations! You have constructed Coulomb's law.
- $\overrightarrow{F_{12}} = \frac{kq_1q_2}{r_{12}^2}\,\hat{r}$



Coulomb's Law (vector form)

- **The force between 2 charges arbitrarily located**



$\hat{r} = \vec{r}/r$ from q_2 to q_1

$$\vec{F}_{21} = \frac{k\, q_1 q_2}{r^2}\hat{r} = \frac{k\, q_1 q_2}{r^3}\vec{r}$$

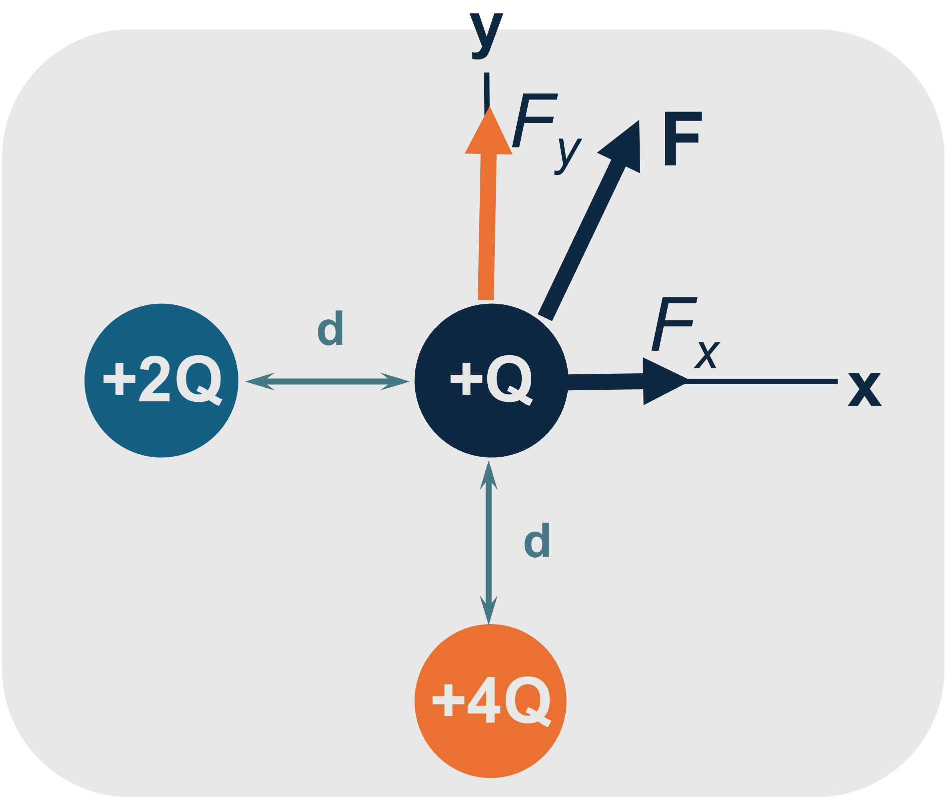
$$= \frac{k\, q_1 q_2}{r^3}\left[(x_1 - x_2)\hat{\imath} + (y_1 - y_2)\hat{\jmath} + (z_1 - z_2)\hat{k}\right]$$

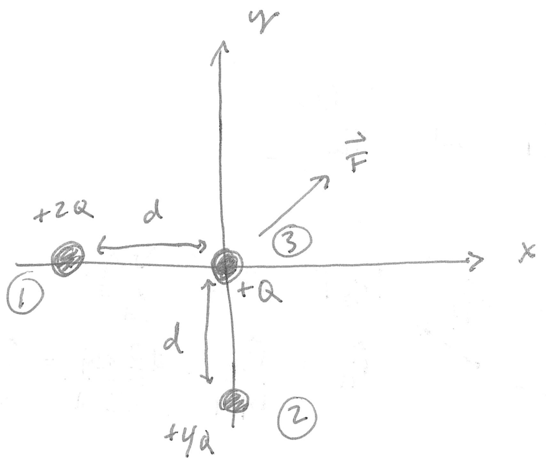
Multiple charges

- Electrostatic forces obey the law of superposition
- If I have many charges q_i , the total force on one of those charges Q can be described as a sum of all the individual forces from individual charge pairs.
- $\vec{F} = kQ \sum_i \frac{q_i}{r_i^2} \hat{r}_i$ where $\hat{r}_i$ is the distance between each of the charges q_i and the charge Q.

We need vectors to solve problems with multiple charges.

Find the direction and magnitude of the net force on charge +Q due to the other two charges.





Find the net (total) force on +Q.

Two sources of interaction:

1-3 , 2-3

$$\vec{F}_{13} = k \frac{q_1 q_3}{r_{13}^2} \hat{r}_{13} \qquad \vec{r}_{13} = \xrightarrow{d} \quad d\hat{x}$$

$$= \frac{k(2Q)(Q)}{d^2} \hat{x}$$

$$\vec{F}_{23} = \frac{k q_2 q_3}{r_{23}} \hat{r}_{23} \qquad \vec{r}_{23} \uparrow \quad d\hat{y}$$

$$= \frac{k(4Q)(Q)}{d^2} \hat{y}$$

$$\boxed{\vec{F}_{tot} = \vec{F}_{13} + \vec{F}_{23} = \frac{kQ^2}{d^2}(2\hat{x} + 4\hat{y})}$$

This is sufficiently defined.

magnitude: $|\vec{F}_{tot}| = \frac{kQ^2}{d^2}\sqrt{2^2+4^2} = \frac{kQ^2}{d^2}\sqrt{20} = \frac{2kQ^2}{d^2}\sqrt{5}$

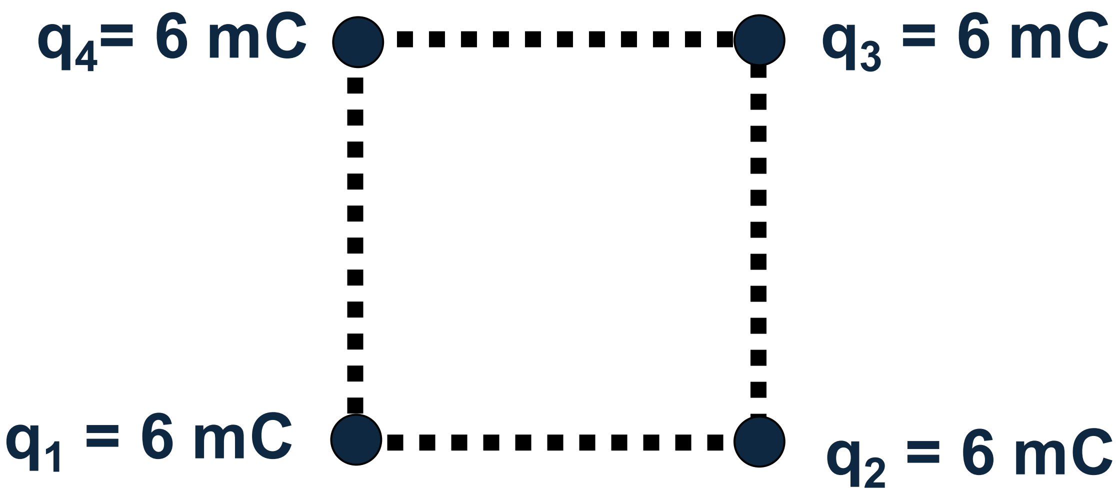
direction: $\hat{F}_{tot} = \frac{\vec{F}_{tot}}{|F_{tot}|} = \frac{\cancel{k}\cancel{Q^2}/\cancel{d^2}}{\sqrt{5}\cdot 2\cancel{k}\cancel{Q^2}/\cancel{d^2}}(2\hat{x}+4\hat{y}) = \frac{1}{\sqrt{5}}\hat{x} + \frac{2}{\sqrt{5}}\hat{y}$

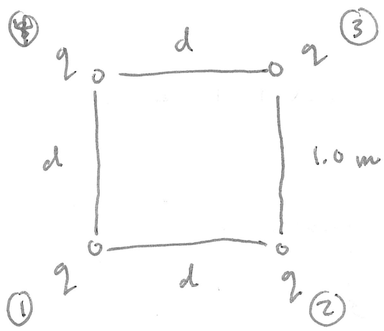
or $\tan\theta = \frac{F_y}{F_x} = \frac{4}{2} = 2$

$\theta = \arctan(2) = 1.1 \text{ rad} = 63.4^\circ$

Problem
Coulomb's Law

- **Charges are placed at each corner of a square 1.0 m on a side. Find the magnitude and direction of the net force on charge q_1 and q_2.**





$q = 6\,mC$ hold onto that for later!

Find ① $F_{net,1}$ and ② $F_{net,2}$

$\vec{F}_{13}$ $\quad |r_{13}| = \sqrt{2}\,d$

$$\hat{r}_{13} = \frac{\vec{r}_{13}}{|r_{13}|} = \frac{d\hat{x} + d\hat{y}}{\sqrt{2}\,d} = \frac{\hat{x}+\hat{y}}{\sqrt{2}}$$

① $$\vec{F}_{net,1} = \vec{F}_{41} + \vec{F}_{31} + \vec{F}_{21}$$

$$= kq^2\left(\frac{1}{d^2}\hat{y} + \frac{1}{2d^2}\cdot\frac{1}{\sqrt{2}}(\hat{x}+\hat{y}) + \frac{1}{d^2}\hat{x}\right)$$

$$= \frac{kq^2}{d^2}\left[\left(1+\frac{1}{2\sqrt{2}}\right)\hat{x} + \left(1+\frac{1}{2\sqrt{2}}\right)\hat{y}\right)\Big]$$

$$\frac{2\sqrt{2}+1}{2\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{4+\sqrt{2}}{4} = 1+\frac{\sqrt{2}}{2} \quad \text{opt.}$$

$$\vec{F}_{net,1} = \frac{kq^2}{d}\left(1+\frac{1}{2\sqrt{2}}\right)(\hat{x}+\hat{y})$$

② $$\vec{F}_{net,2} = \vec{F}_{12} + \vec{F}_{42} + \vec{F}_{32}$$

we could do the whole thing again

BUT this problem is very symmetric.

$\vec{F}_{net,1} = \uparrow + \nearrow + \rightarrow = \nearrow$

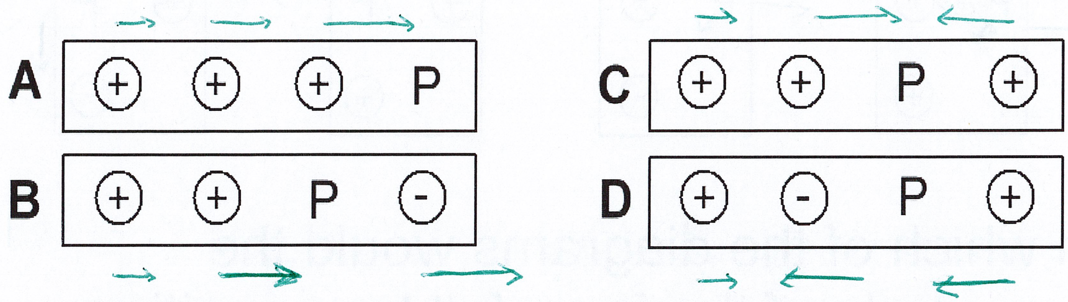
should have same magnitude

$\vec{F}_{net,2} = \leftarrow + \nwarrow + \uparrow = \nwarrow \sim -\hat{x} + \hat{y}$

$$\vec{F}_{net,2} = \frac{kq^2}{d^2}\left(1+\frac{1}{2\sqrt{2}}\right)(-\hat{x}+\hat{y})$$

Name: ______________________________

In the figure below are shown four one-dimensional arrangements of charge. Each of the charges has the same magnitude, but some are positive and some are negative. In each diagram a point is labeled "P". All distances are to the same scale.

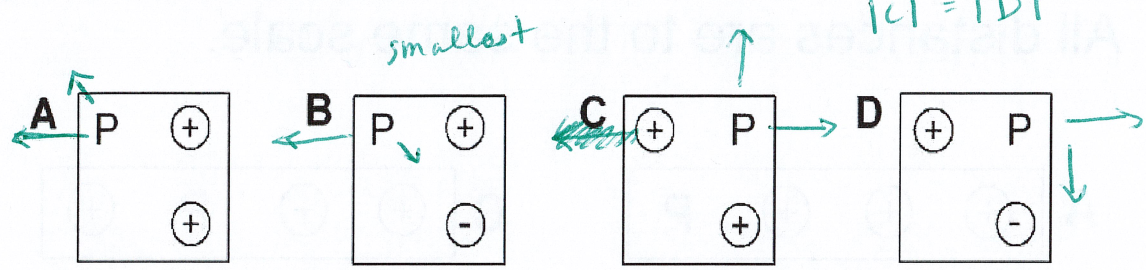


In which of the diagrams would the magnitude of the force felt by a positive test charge placed at P be the largest?

1. A
2. B
3. C
4. D
5. A and B
6. A and C
7. B and D
8. All are equal
9. can't tell

B has the most + biggest arrows pointing in the same direction.

In the figure below are shown two two-dimensional arrangements of charge. Each of the charges has the same magnitude, but some are positive and some are negative. In each diagram a point is labeled “P”.



In which of the diagrams would the magnitude of the force felt by a positive test charge placed at P be the largest?

1. A
2. B
3. C
4. D
5. A and B
6. A and C
7. B and D
8. All are equal
9. can’t tell

(c)

this one is slightly larger
(A)