Capacitor Energy

PHY 114-01

Fall 2026

Announcements

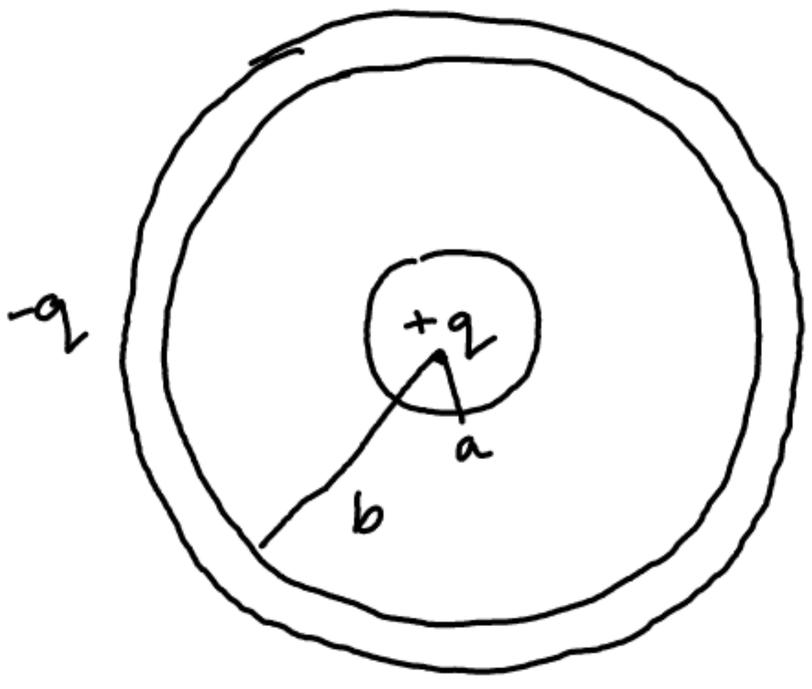
- Midterm reflections due tonight 11:59pm
  - If you need more time, ask!
- Midterm reporting deadline Friday
- Alternative grading schemes
  - If you do better on midterm 2 than midterm 1: replace midterm 1 score with average of midterm 1 and midterm 2.
  - Goal: grade reflects what you know by the end of the semester, not what you know at the beginning of the semester.
- HW4 had an error. If you see another one, let me know!

Agenda

- Last time:
  - Capacitors and capacitance

- This time:
  - Review capacitance
  - Capacitor energy storage
  - Dielectrics

Spherical Capacitor



Two conducting spherical shells

radius a, charge +q

radius b, charge -q

Find $C = \frac{Q}{V}$

Q is just q

V needs to be calculated from $\vec{E}$.

$\vec{E}$ in the region $a < r < b$ can be found using Gauss's Law:

$$E \cdot A = q_{enc} / \varepsilon_0$$

$$E \cdot 4\pi r^2 = q / \varepsilon_0$$

↑ surface area of Gaussian surface

↖ charge enclosed by surface

$$\vec{E} = \frac{q}{4\pi\varepsilon_0} \frac{1}{r^2} \hat{r}$$

Then, $V_{ab} = -\int_a^b \vec{E} \cdot d\vec{r}$

$$= -\int_a^b \frac{q}{4\pi\varepsilon_0} \frac{1}{r^2} \, dr$$

$$V_{ab} = \frac{-q}{4\pi\varepsilon_0} \frac{1}{r}\Big|_a^b = \frac{-q}{4\pi\varepsilon_0} \left(\frac{1}{b} - \frac{1}{a}\right)$$

If $C = Q/V$,

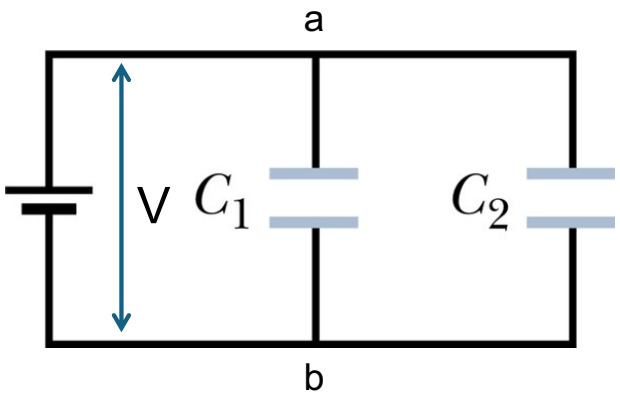
$$C = \frac{q}{V_{ab}} = \frac{-q}{\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{b} - \frac{1}{a}\right)}$$

$$\boxed{C = \frac{4\pi\varepsilon_0}{\left(\frac{1}{a} - \frac{1}{b}\right)}}$$

Capacitors in Parallel

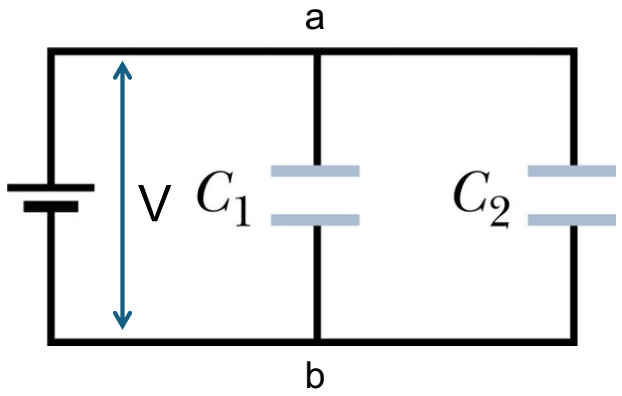
Here is a circuit with two capacitors connected together in parallel. What is the equivalent capacitance?

- **potential difference between points a and b is the same for the capacitors, across the same battery: they are said to be in parallel**
  - $V_1 = V_2 = V$
- **charges on the capacitors are**
  - $Q_1 = C_1V_1 = C_1V$
  - $Q_2 = C_2V_2 = C_2V$
- **total charge on both capacitors is**
  - $Q = Q_1+Q_2 = (C_1 + C_2)V$



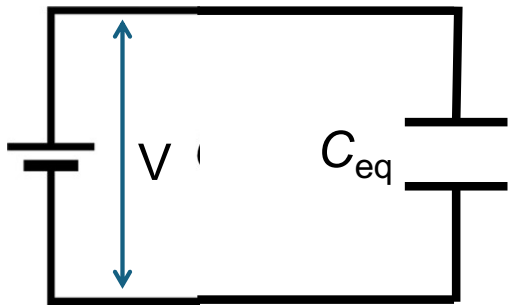
Capacitors in Parallel

- **total charge on both capacitors is**
  - **$Q = Q_1 + Q_2 = (C_1 + C_2)V$**



- **the equivalent capacitor is C_{eq}, with the equivalent capacitance**
  - **$C_{eq} = Q/V = C_1 + C_2$**

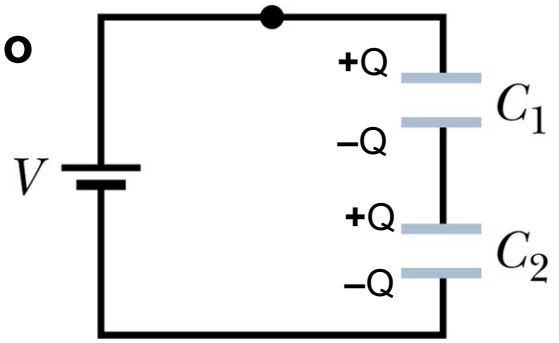
Net capacitance of capacitors in parallel is additive; C_{eq} is larger than any single C_i



equivalent circuit

Capacitors in Series

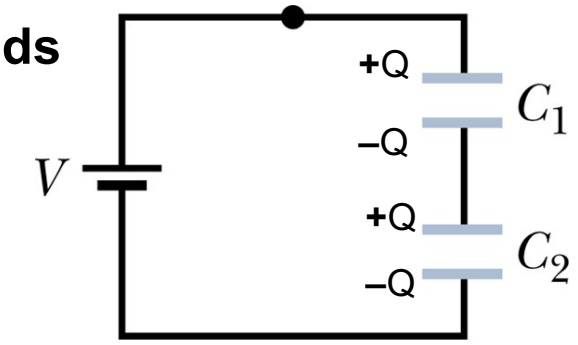
Here is a circuit with two capacitors connected together in series. What is the equivalent capacitance?



- **the charge flows in a single path, so it is the same for both capacitors: they are said in series**
  - **$Q_1 = Q_2 = Q$**
- **the voltage on the capacitors are**
  - **$V_1 = Q_1/C_1 = Q/C_1$**
  - **$V_2 = Q_2/C_2 = Q/C_2$**
- **total voltage across capacitors adds**
  - **$V = V_1+V_2 = Q(1/C_1 + 1/C_2)$**

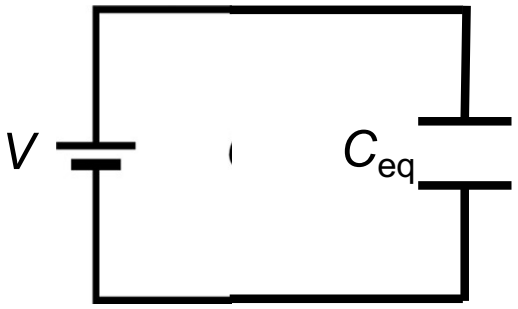
Capacitors in Series

- **total voltage across capacitors adds**
  - **$V = V_1 + V_2 = Q(1/C_1 + 1/C_2)$**



- **the equivalent capacitor is C_{eq}, with the equivalent capacitance**
  - **$V/Q = 1/C_{eq} = 1/C_1 + 1/C_2$**

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$



equivalent circuit

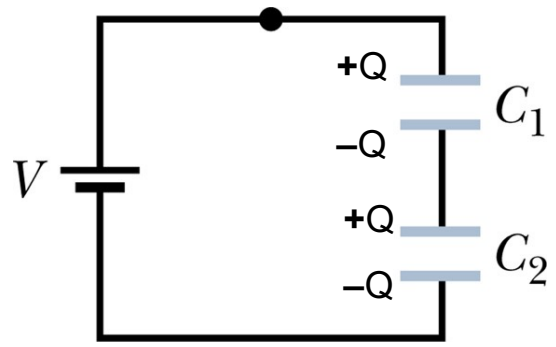
Reciprocal of net capacitance of capacitors in series is the sum of the reciprocals; C_{eq} is smaller than any single C_i .

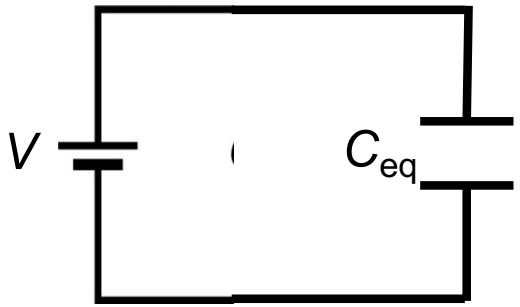
Capacitors in Series

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

$$= \frac{C_1}{1 + C_1 / C_2} = \frac{C_2}{1 + C_2 / C_1}$$

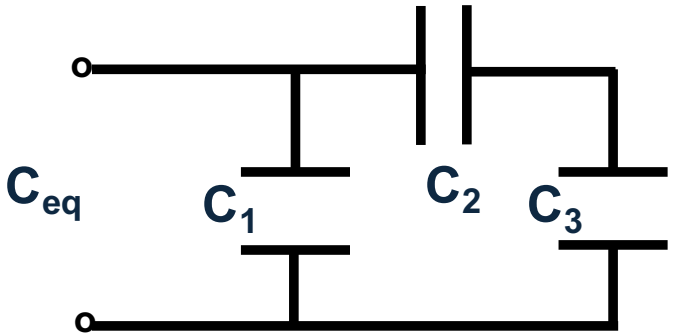




equivalent circuit

C_{eq} is smaller than the smallest C_i. For n identical capacitors in series, $C_{eq} = C/n$

ConceptCheck Capacitors in Circuits



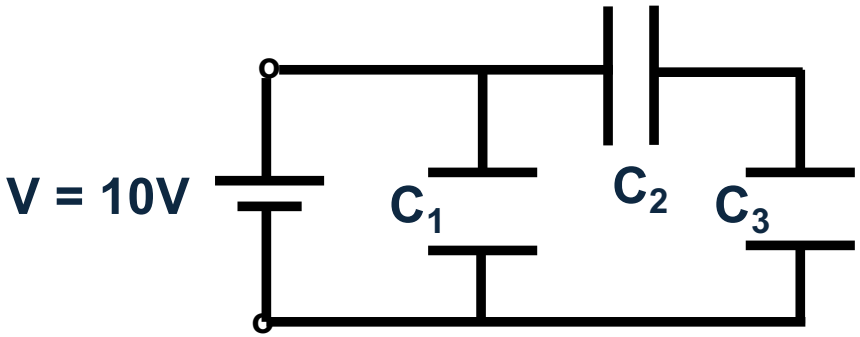
If all the capacitors have the same value C, what is the equivalent capacitance C_{eq} of this circuit ?

A. 3C/2
B. 2C
C. 3C
D. C/2
E. C/3

C_2 and C_3 are in series, giving C_{23} = ½ C; which in turn is in parallel with C_1; so C_{eq} = 3C/2

ConceptCheck

Capacitors in Circuits



How does the voltage V_1 across the first capacitor C_1 compare to the voltage V_2 across the second capacitor C_2 ?

A. $V_1 = V_2$

B. $V_1 > V_2$

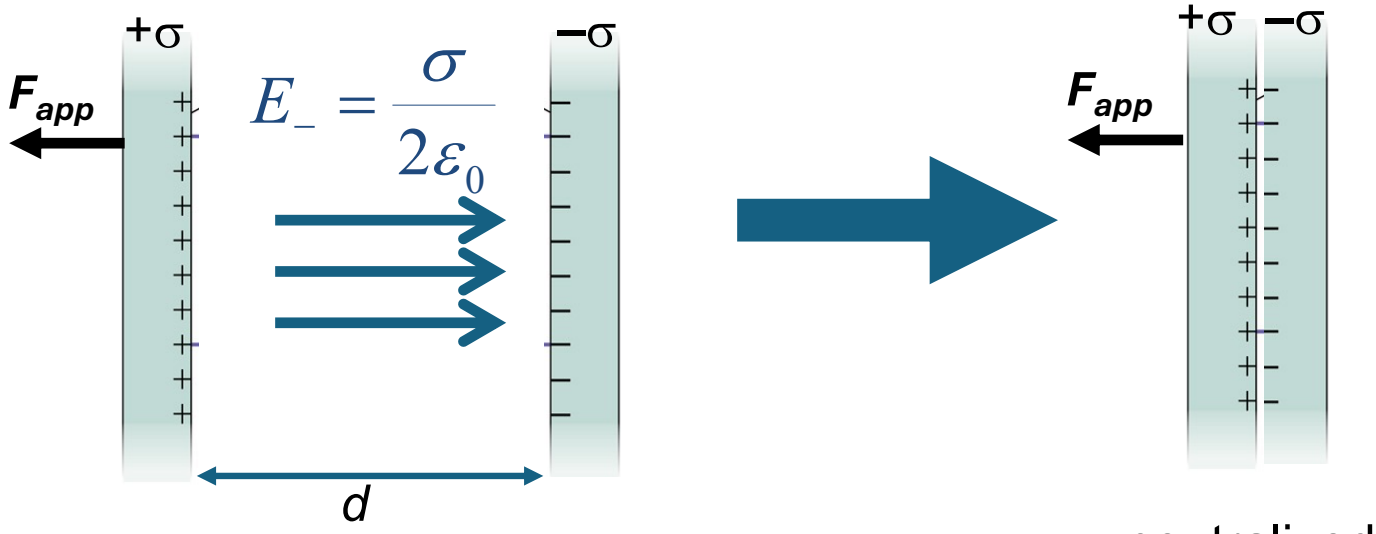
C. $V_1 < V_2$

D. **not enough info**

C_2 and C_3 share V, while V_1=V

Energy Stored in a Capacitor

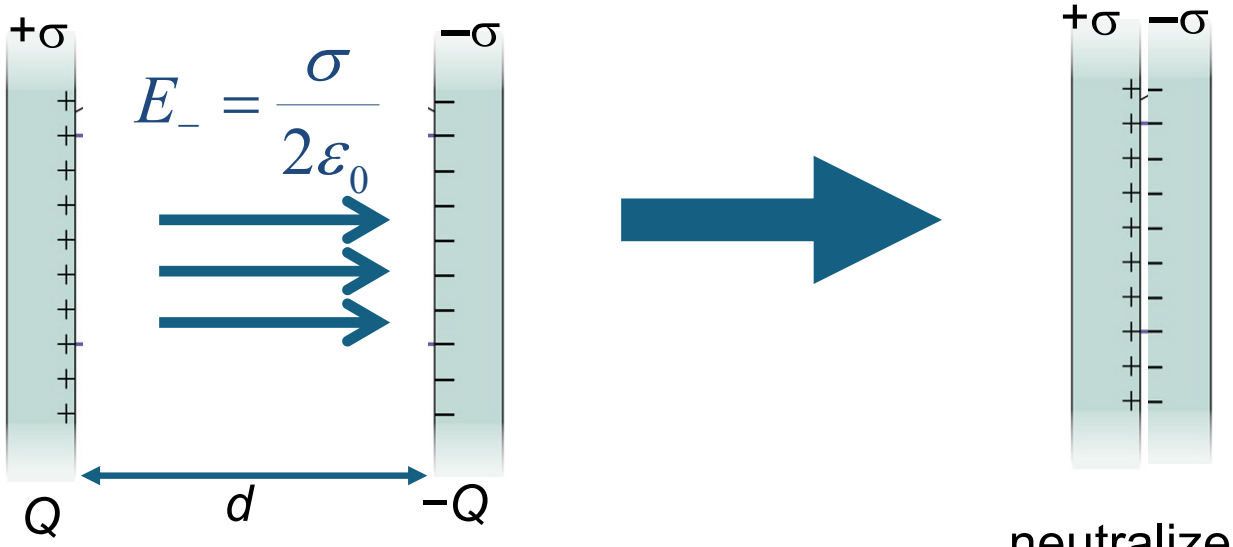
Key idea: The positive plate is in the electric field E_- of the **negative plate**. If the positive plate were free to move, it would crash into the negative plate, releasing energy



neutralized

To prevent it from crashing, we do work holding it back; and the **amount of work = energy stored=U_f-U_i**

Energy in a Capacitor, algebraic way



$$W = Fd = QE_-d = \frac{Q\sigma d}{2\varepsilon_0} = \frac{Q}{2}\frac{\sigma A}{\varepsilon_0 A/d}, \text{ but, } \sigma A = Q,\ C = \frac{\varepsilon_0 A}{d}$$

$$W = \frac{1}{2}\frac{Q^2}{C} = U$$ energy stored in a capacitor

Energy Stored in a Capacitor

$$U = \frac{1}{2}\frac{Q^2}{C}$$

Since $Q = CV$

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$