Electric Potential and Electric Field

PHY 114-01

Fall 2026

Agenda

- Last time:
  - Relationship between electric field and electric potential
  - Calculating electric potential

- This time:
  - Wrap up calculating field and potential
  - Equipotential surfaces

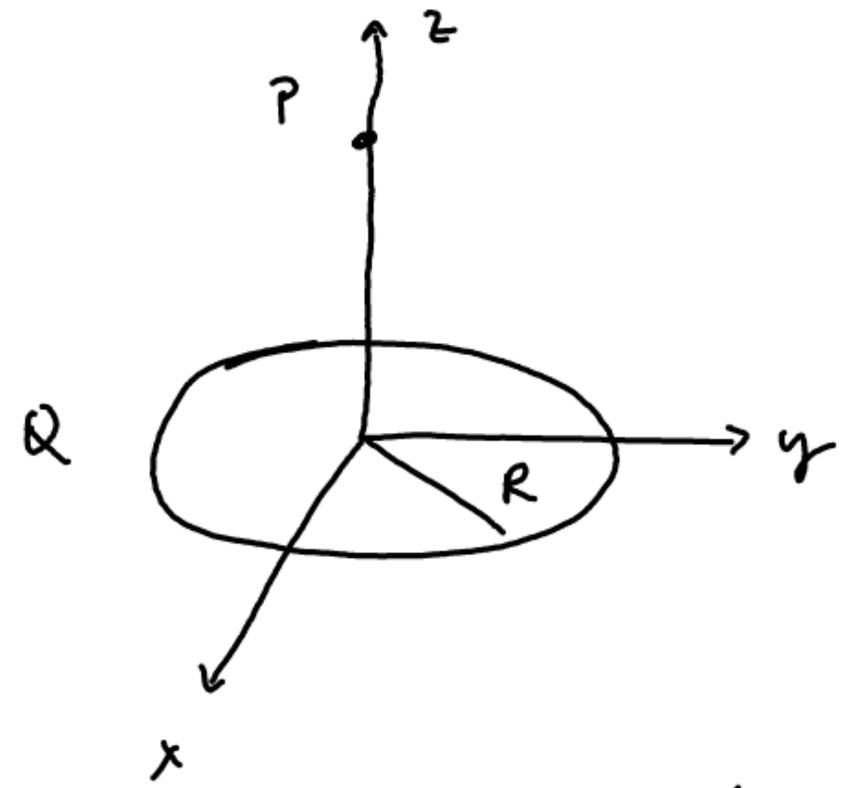
Get field from potential

- It can be easier to calculate the potential of a charge distribution because you are just adding scalar values.
- Then, take a gradient to get the field.

- Example: ring of uniform charge potential

Ring of Uniform Charge

We already did this for electric field. Let's see what happens when we start with electric potential instead.



integrate over all tiny contributions

$$dV = \frac{dq}{4\pi\varepsilon_0 r}$$

advantages: no vectors

$$V = \int_0^{2\pi} \frac{(Q/2\pi R)\cdot R\,d\theta}{4\pi\varepsilon_0\sqrt{R^2+z^2}}$$

$$= \frac{Q}{2\pi\cancel{R}}\,\frac{\cancel{R}}{4\pi\varepsilon_0\sqrt{R^2+z^2}}\int_0^{2\pi} d\theta$$

$$= \frac{Q}{\cancel{2\pi}}\cdot\cancel{2\pi}\cdot\frac{1}{4\pi\varepsilon_0\sqrt{R^2+z^2}}$$

$$\boxed{V = \frac{Q}{4\pi\varepsilon_0\sqrt{R^2+z^2}}}$$

like a point charge!

If we use $\vec{E} = \vec{\nabla} V$, we should recover the result we previously found for electric field.

$$\vec{E} = -\vec{\nabla} V = -\frac{\partial}{\partial z} V \hat{z} \quad \leftarrow V \text{ only depends on } z$$

$$= -\frac{\partial}{\partial z}\left[\frac{Q}{4\pi\varepsilon_0} \cdot \frac{1}{\sqrt{R^2+z^2}}\right] \hat{z}$$

$$= -\frac{Q}{4\pi\varepsilon_0} \cdot \left(-\frac{1}{2}\right) \frac{2z}{(R^2+z^2)^{3/2}}$$

$$\vec{E} = \frac{Q z}{4\pi\varepsilon_0 (R^2+z^2)^{3/2}} \hat{z}$$

matches our previous result

with less effort!

↑

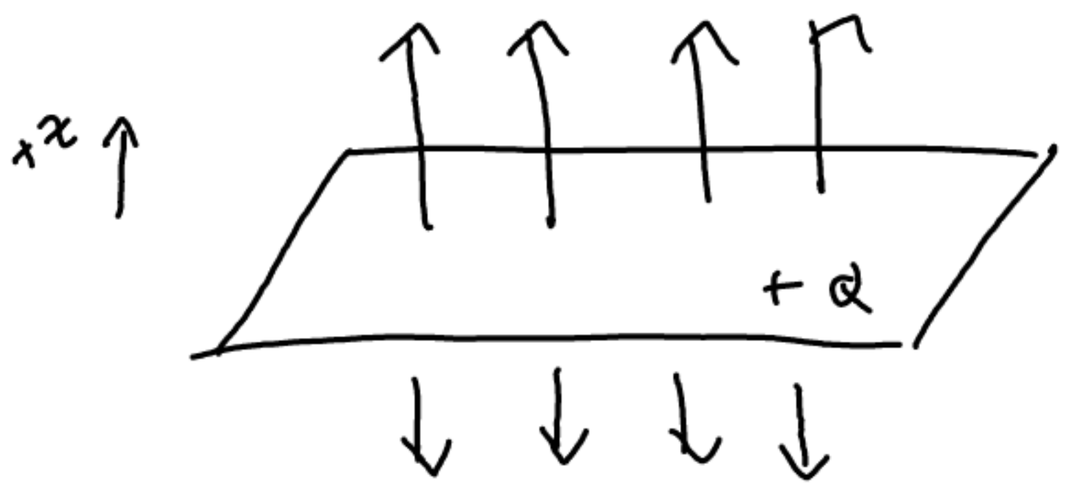
no vectors

Potential from a distribution of charges

- Option 2: Integrate the field
  - First find the field, and then integrate to get the potential
  - This will be useful in situations where we have previously calculated the field or when we can easily find it through Gauss's Law

- Example: plane of charge

Parallel Plates

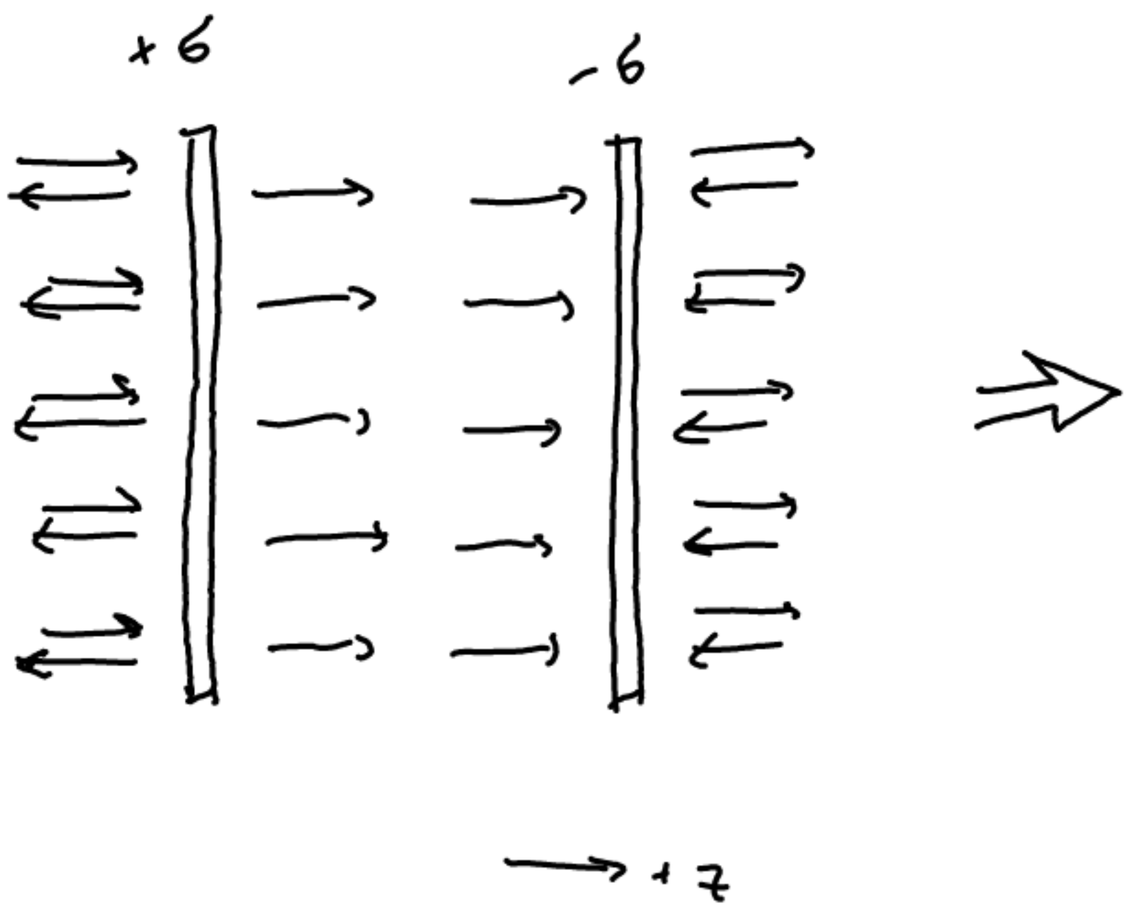


from Gauss's Law, we know the field of a plane w/ charge Q:

- the charge is distributed evenly over each surface of the plane with density σ

- Gauss's Law: $E \cdot 2A = \frac{\sigma \cdot A}{\varepsilon_0}$

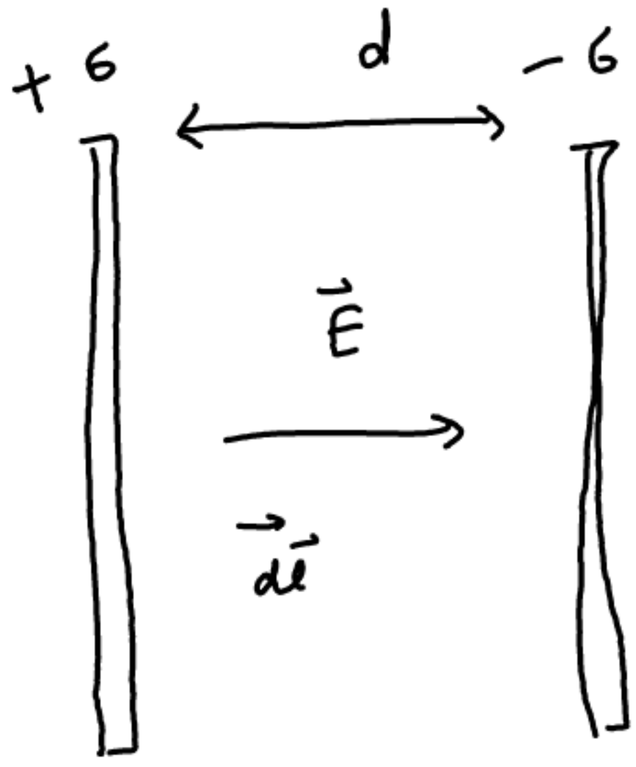
$$E = \frac{\sigma}{2\varepsilon_0}\hat{z}$$



$$E = \frac{\sigma}{\varepsilon_0}\hat{z}$$

between the plates

Now we can find V from $\vec{E}$:



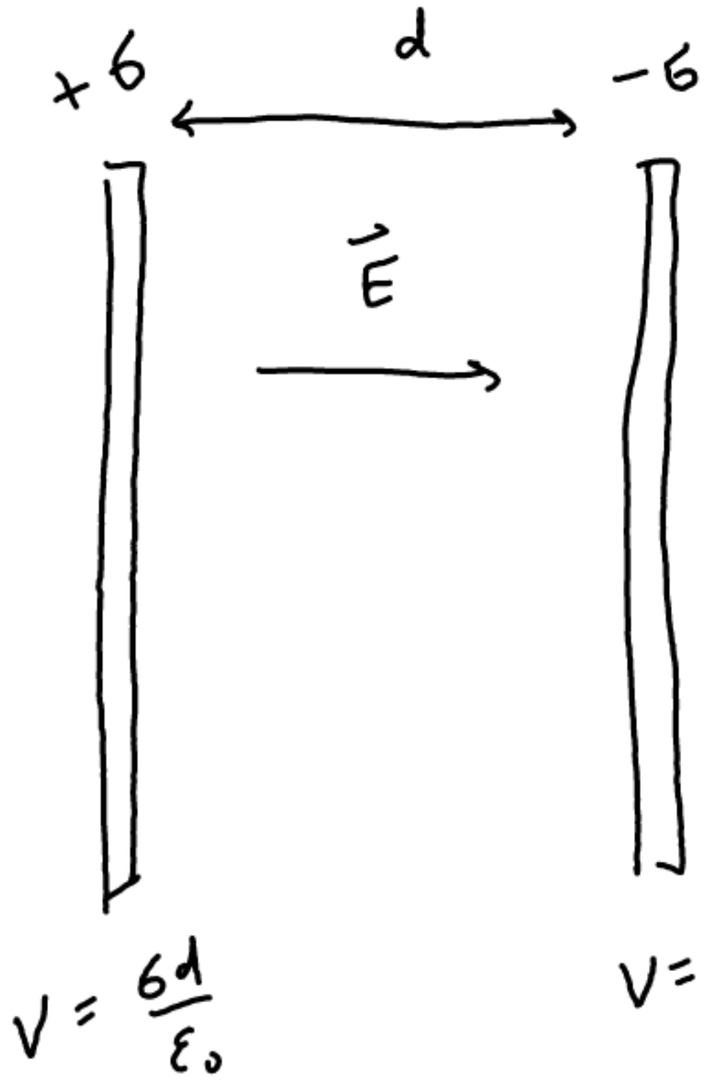
$$\Delta V = - \int \vec{E} \cdot d\vec{l}$$

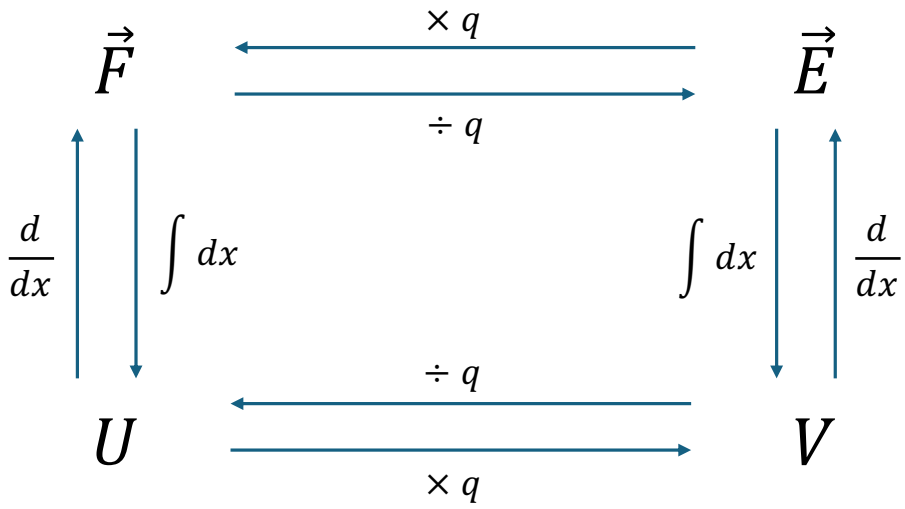
$$= - E \cdot d$$

$$\Delta V = - \frac{6}{\varepsilon_0} d$$

go from + 6 plate to - 6 plate

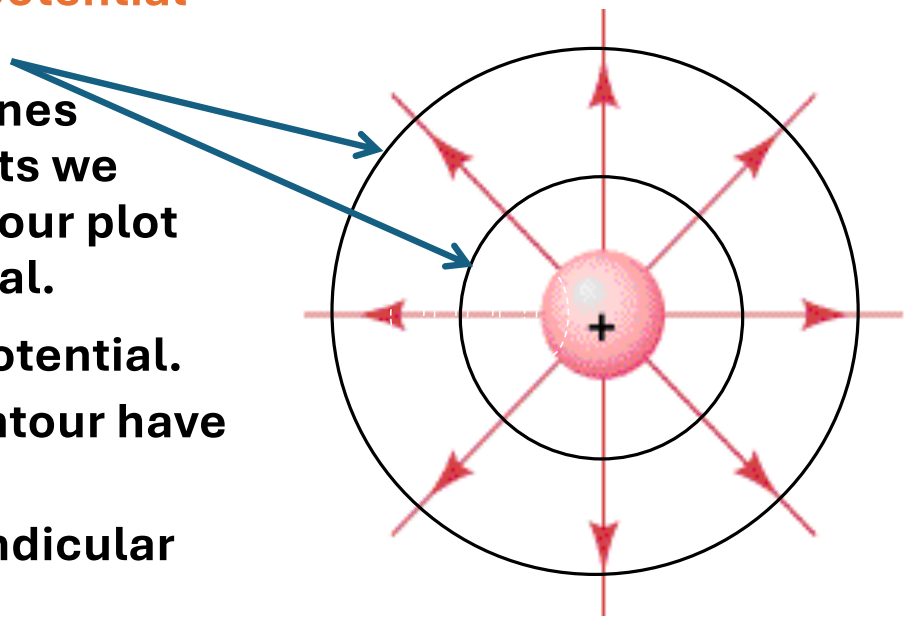
potential <u>drops</u> by $6/\varepsilon_0$



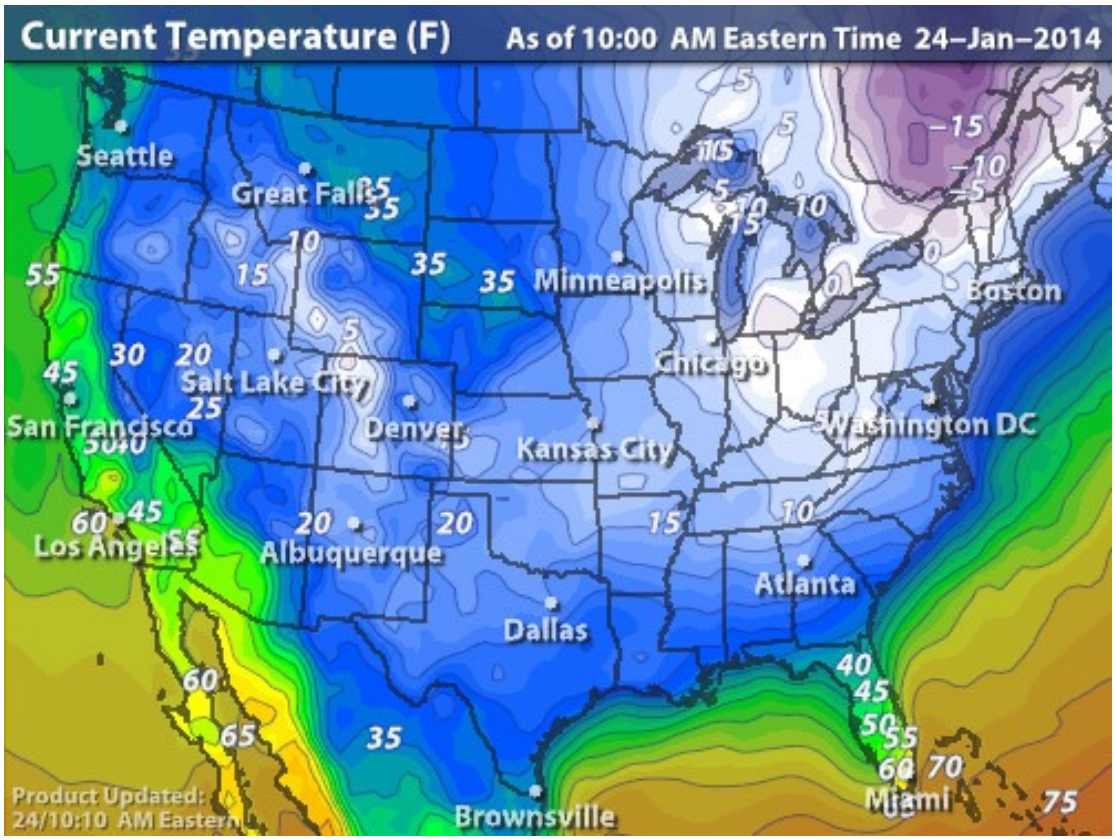

$\vec{F}$
$\times q$
$\div q$
$\vec{E}$
$\frac{d}{dx}$
$\int dx$
$\int dx$
$\frac{d}{dx}$
$\div q$
$\times q$
U
V

Equipotential Surfaces

- **adjacent points at the same potential form an equipotential surface.**
  - **if we were to draw lines between these points we would create a contour plot showing the potential.**
- **contours of constant potential.**
  - **all points on the contour have the same value of V.**
  - **contours are perpendicular to electric fields**



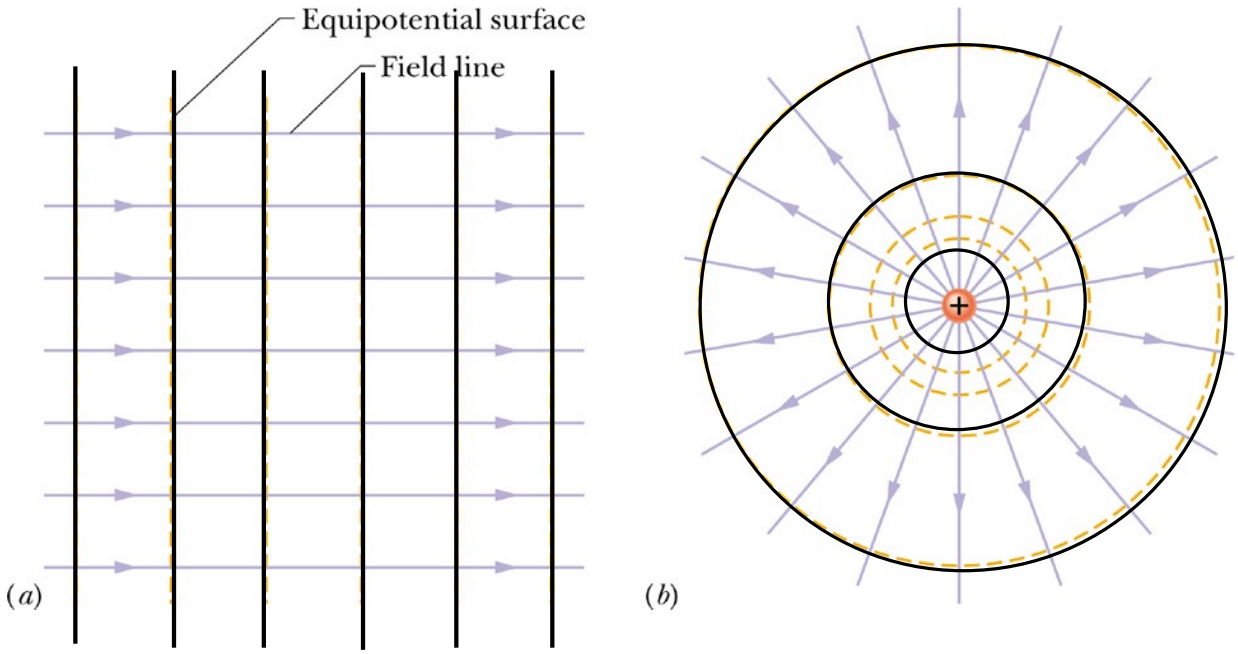
Equipotential Surfaces



Lines on this map are contours of equal temperature

Equipotential Surfaces

- **Equipotential surface**
  - **contours of constant potential**

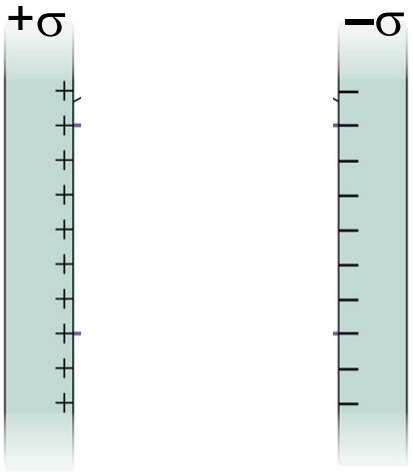


uniform electric field **point charge**

Activities

- **Draw electric field lines of a dipole and of parallel plates**
- **Draw contour lines of equipotential surfaces**

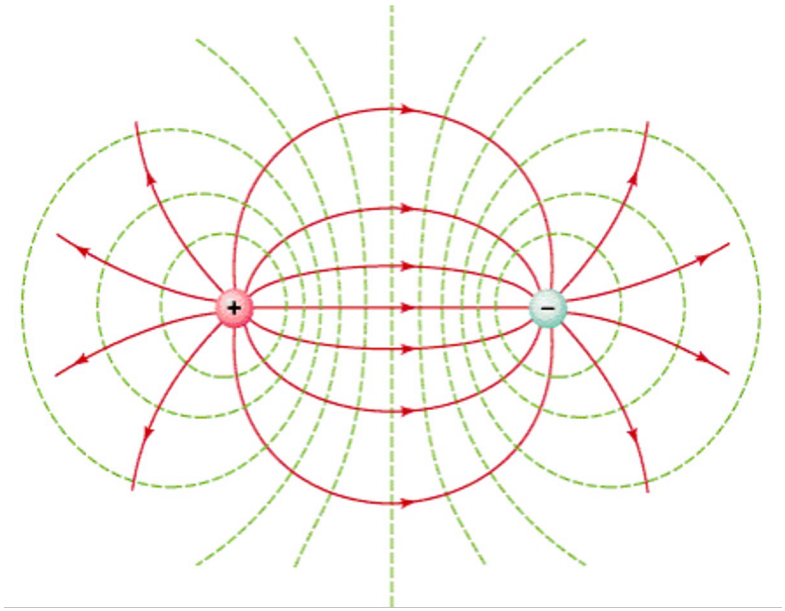




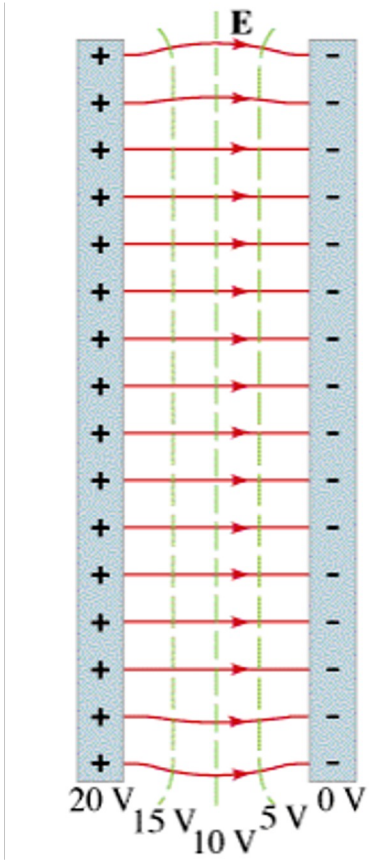
Equipotential Surfaces

- **Equipotential surface**
  - **contours of constant potential**

An electric dipole



parallel plates



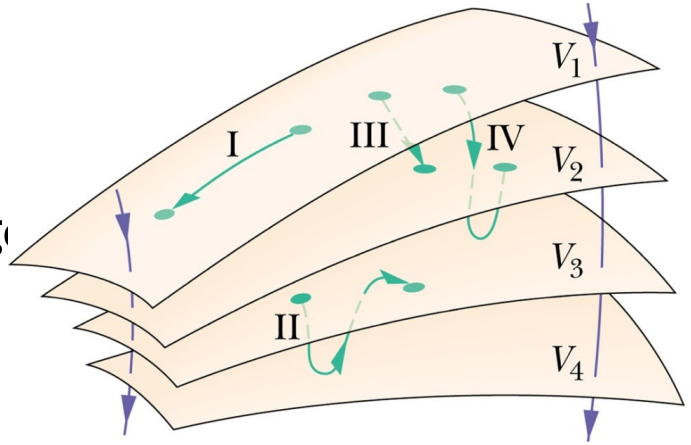
Equipotential Surfaces

- **Equipotential surface**
  - **contours of constant potential**

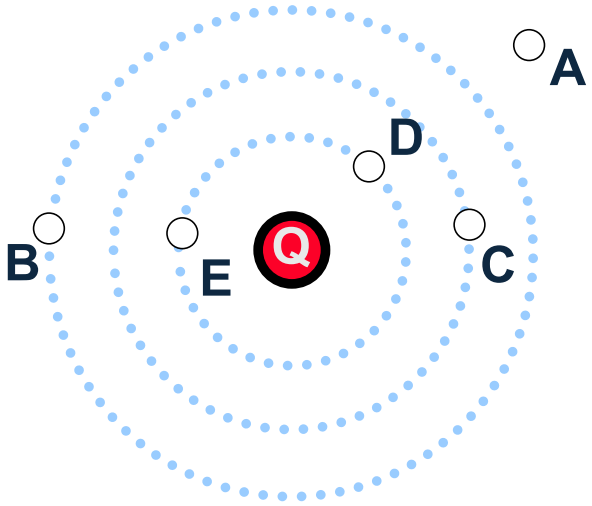
- **No work is required to move charg along an equipotential surface**
  - **Since work done is:**

  $W = -\Delta U = -q\,\Delta V = -q(V_b - V_a)$

    - **if there is no change in potential, $V_b = V_a$**
    - **no work is done on the charge**



Concept Check Equipotential Surfaces



Which two points have the same potential ?

A. A and D
B. B and E
C. B and C
D. D and E
E. no pair

They are on the same equipotential surface