Conductors in Electrostatic Equilibrium

PHY 114-01

Fall 2026

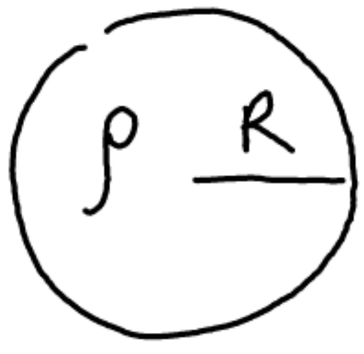
Agenda

- Last time:
  - Gauss' law for infinite line and plane
- This time:
  - Gauss' Law for a solid sphere
  - Conductors in electrostatic equilibrium

Solid sphere of charge with Gauss' Law

- Outside the sphere
- Inside the sphere

Sphere of charge



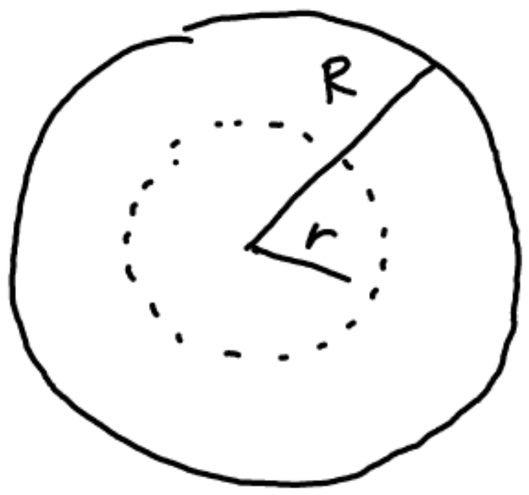
uniform charge density ρ

Find $E(r)$ for <u>all</u> r

1. spherical symmetry

2.

$r < R$



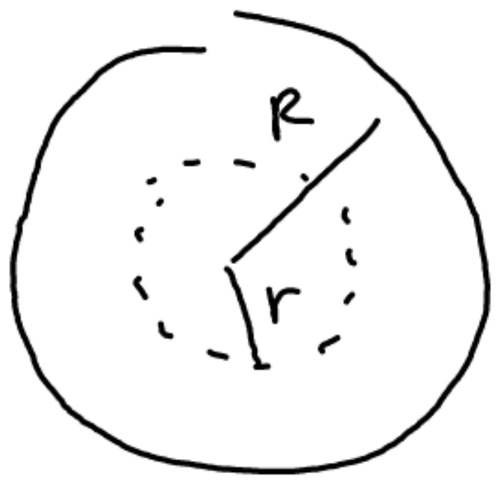
Gaussian surface inside sphere

$r = R$ Gaussian surface at charged sphere surface

$r > R$



$r < R$



$$Q \rightarrow \rho = \frac{Q}{V}$$

$$= \frac{Q}{\frac{4}{3}\pi R^3}$$

$$EA = q_{enc} / \varepsilon_0$$

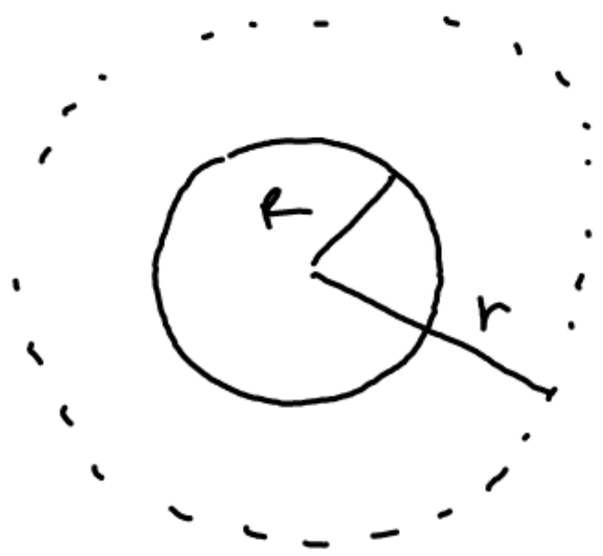
A Gaussian surface $= 4\pi r^2$

$$q_{enc} = \rho V_{enc} = \rho \cdot \frac{4}{3}\pi r^3$$

$$E \cdot 4\pi r^2 = \left(\frac{Q}{\frac{4}{3}\pi R^3} \right) \frac{4}{3}\pi r^3 / \varepsilon_0$$

$$\vec{E}(\vec{r}) = \frac{Q\ r}{4\pi R^3 \varepsilon_0}\ \hat{r}$$

$r \geq R$

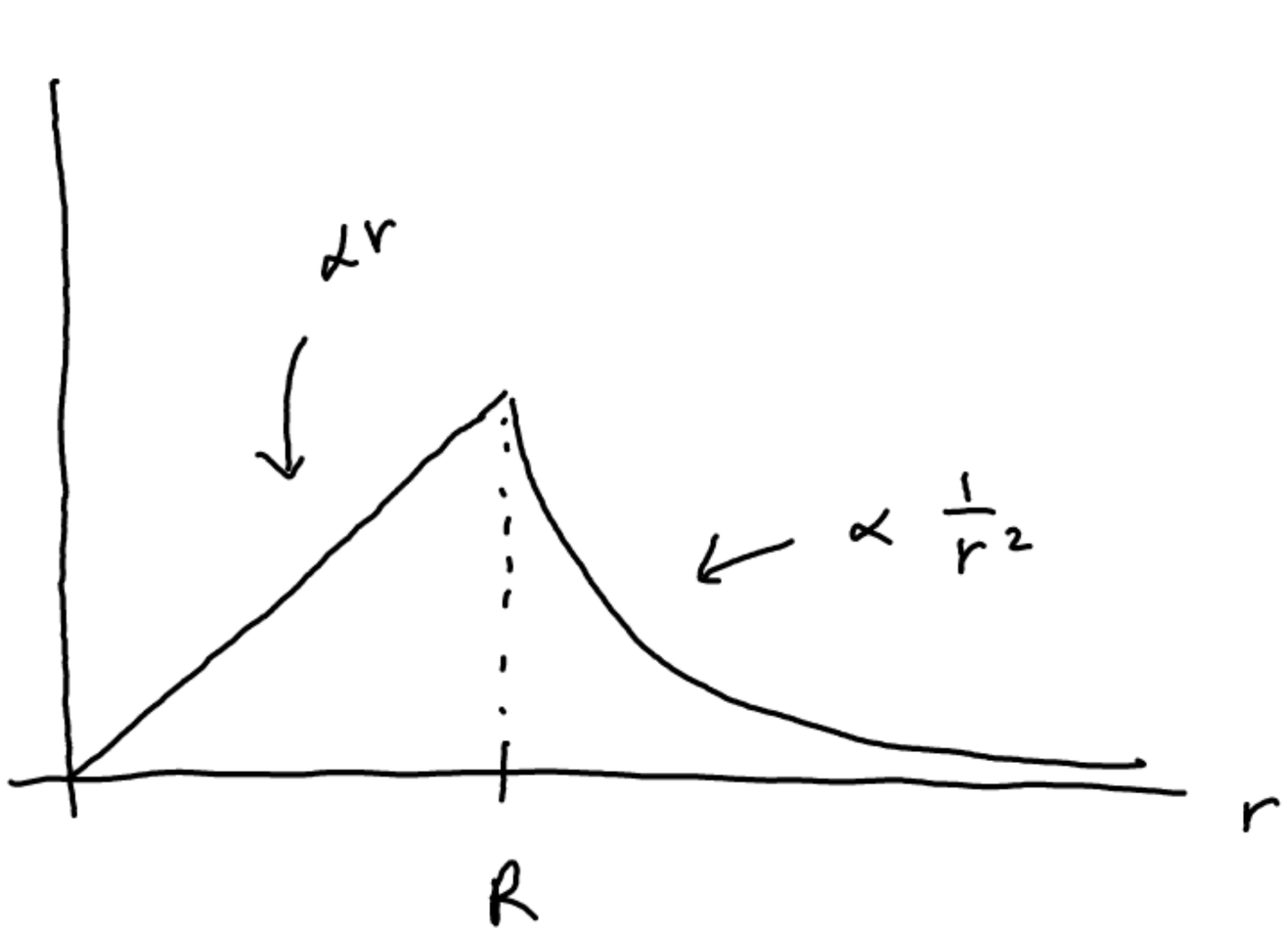


Now, $q_{in} = Q$

$$E A = q_{in} / \varepsilon_0$$

$$E \cdot 4\pi r^2 = Q / \varepsilon_0$$

$$\vec{E} = \frac{Q}{4\pi r^2 \varepsilon_0} \hat{r}$$



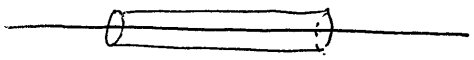
Lecture 12: Gauss' Law and Symmetry

Name: _____

This is a pop quiz! You will get 1 points for being here and turning it in, and 0.2 points for each correct answer for 2 points total. I will give you five minutes to work alone, two minutes to discuss, and two minutes to revise.

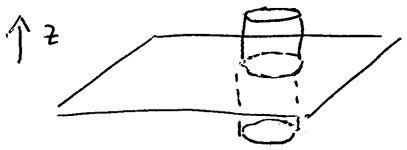
For the following charge distributions, identify the correct symmetry and Gaussian surface. Write the expected form of the electric flux as E (as a function of some coordinate) times some surface area. You may need to add multiple surfaces.

1. An infinite line of charge cylindrical



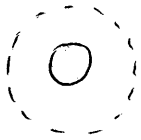
$\Phi_E = E \cdot A = E(r) \cdot 2\pi r l$

2. An infinite plane of charge planar



$\Phi_E = E \cdot A = E(z) \cdot 2A$

3. A sphere of charge spherical



$\Phi_E = E \cdot A = E(r) \cdot 4\pi r^2$

4. A very long line of charge that you are close to

That's approximately an infinite line!

5. A short line of charge that you are far away from

If we're REALLY far away, we can treat it like a point charge ← spherical

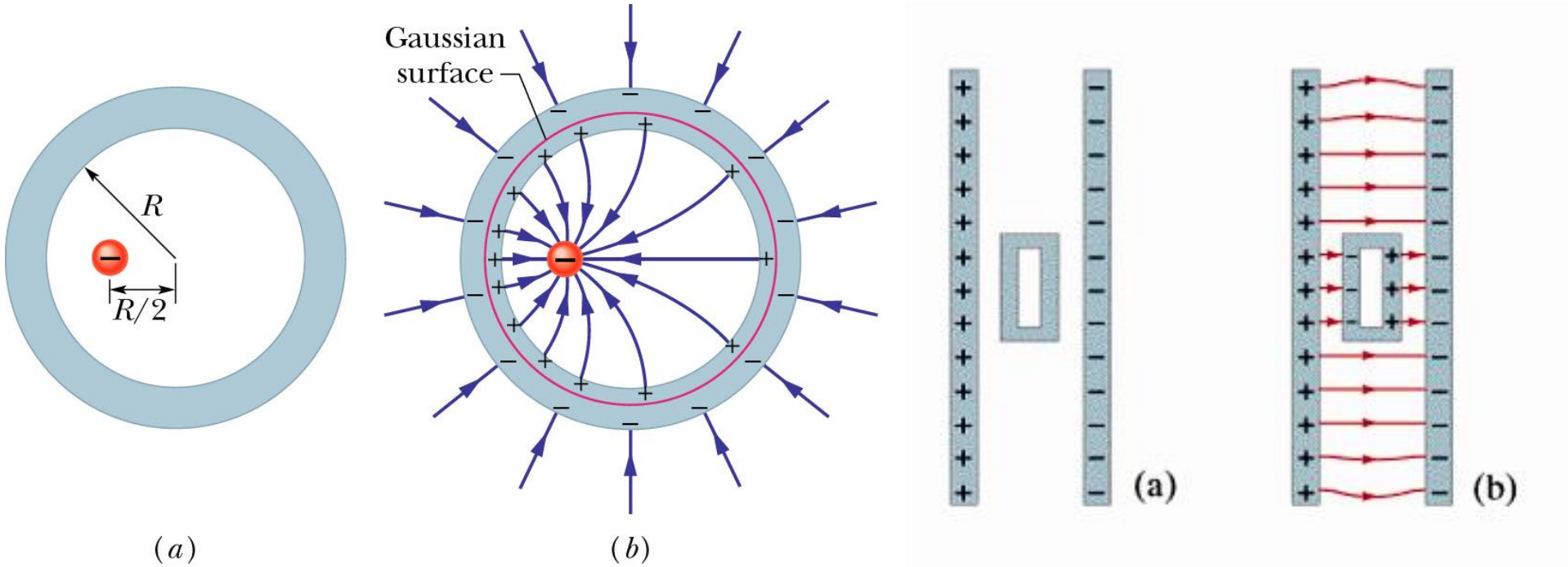
Conductors in Electrostatic Equilibrium

charges can move freely

no motion of charges

- **Electric field is zero everywhere inside the conductor**
  - **if $E \neq 0$, then charges would move -- no equilibrium!!**
- **Excess charge on isolated conductor resides only on the surface of the conductor**
  - **mutual repulsion pushes the charges apart**
- **Electric field is perpendicular to the surface of a conductor**
  - **if a parallel component existed, charges would move!!**
- **For irregular shaped conductors, more charge accumulates near sharp points, i.e. the field strength is greater there**

Electric Field in a Conductor



The electric field inside a conductor is zero; such a conductor is called a Faraday cage

ConceptCheck Conductors

A conductor has a net charge Q of + 3.2 μC. Inside the conductor there is a cavity with a point charge q = –1.2 μC. What is the charge on the inner wall of the cavity?

A. +2.0 μC

B. –2.0 μC

C. +1.2 μC ✓

D. –1.2 μC

E. it depends on the shape of the cavity

The charge on the interior cavity wall must cancel the charge inside the cavity (this ensures there is no electric field within the conductor).

q_{wall} = + 1.2 μC

ConceptCheck Conductors

A conductor has a net charge Q of + 3.2 μC. Inside the conductor there is a cavity with a point charge q = –1.2 μC. What is the charge on the outer wall of the cavity?

A. +2.0 μC ✓

B. –2.0 μC

C. +1.2 μC

D. –1.2 μC

E. it depends on the shape of the cavity

The total charge on the exterior must be the total charge within:
Q + q = 2 uC

Gauss’s law with concentric conducting shells