Using Gauss' Law

PHY 114-01

Fall 2026

Midterm logistics

- In-class 50 minute exam
- Arrive before 8am to get the full exam time!
- “Cheat sheet”
  - 1 page single sided, handwritten
  - Turn it in with your exam. 10% penalty for going over page limit or typed/copied notes.
- 25% multiple choice conceptual, 75% quantitative
- Practice problems:
  - Review homework and lecture quizzes
  - Openstax problems (see chapter review)
  - Halliday and Resnick (library, Physics department)

Agenda

- Last time:
  - Calculating flux
  - Gauss' Law
  - Field from a point charge (again)

- This time:
  - Field from an infinite line of charge
  - Field from an infinite sheet of charge
  - Field from a sphere of charge

Gauss' Law

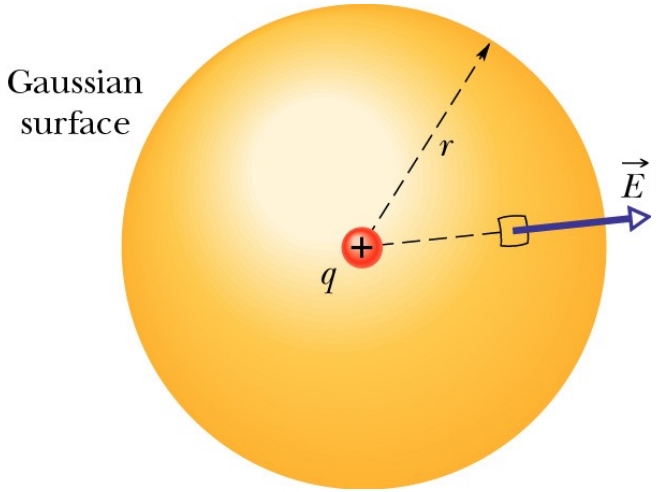
$$\Phi_E = q_{in}/\epsilon_0$$

The flux of the electric field through a closed surface | is equal to | The charge enclosed by that surface (divided by ϵ_0)

- We call the closed surface a "Gaussian surface"
- The surface can be anything as long as it's closed
- We will chose surfaces that are
  - 1. matching the symmetry of our charge distribution
  - 2. easy to calculate surface area for

Gauss' Law

$$\Phi_E = \oint \vec{E} \cdot \overrightarrow{dA}$$



- Integral over the closed surface
- For every tiny bit of surface $d\vec{A}$ is the surface area element times the local normal vector.

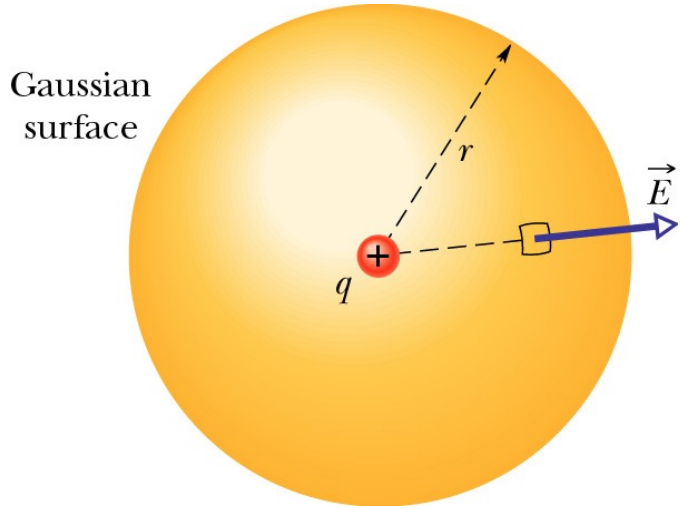
For the sphere around a point charge, E and A are automatically parallel

$$q = \varepsilon_0 \oint E dA$$

$$q = \varepsilon_0 E(4\pi r^2)$$

Gauss' Law & Coulomb's Law

Consider a point charge q surrounded by a spherical Gaussian surface of radius r



- **Gauss' Law can be written as**

$$q = \varepsilon_0 E(4\pi r^2)$$

- **rearranging this gives**

Coulomb's Law

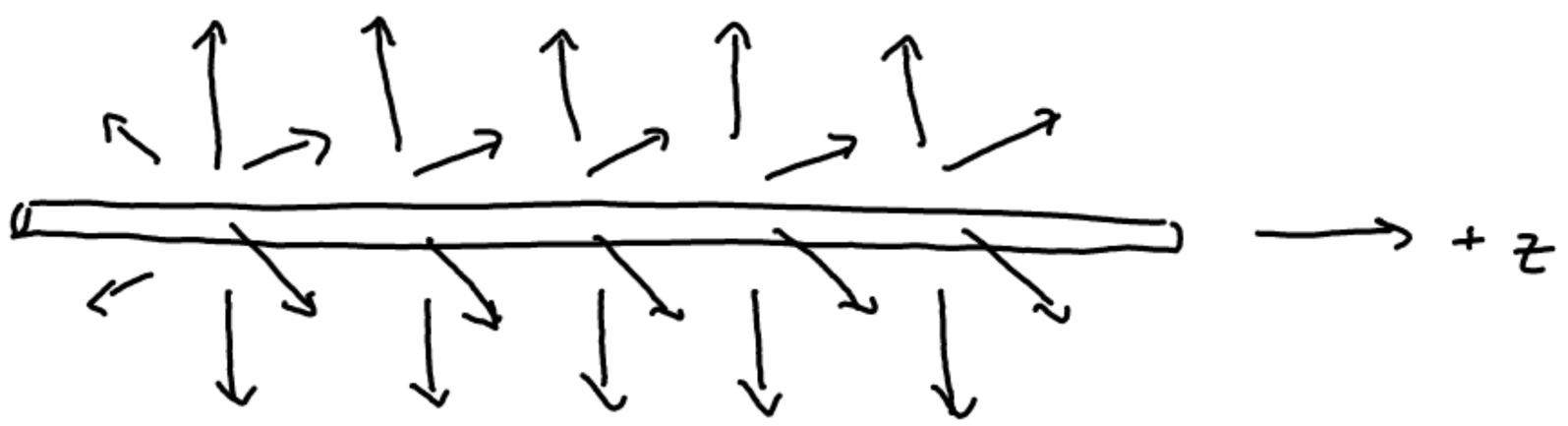
$$E = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q}{r^2} \quad \textbf{OR} \quad E = k\frac{q}{r^2}$$

Procedures for using Gauss' Law

1. Identify the symmetry of your charge distribution.
2. Draw a Gaussian surface with that symmetry around the charges. E should be normal to the surface.
3. Write Gauss' Law $EA = \frac{q_{in}}{\epsilon_0}$. Note that this form only works if you were successful in part 2.
    1. Solve the left side using the surface area of your Gaussian surface
    2. Solve the right side by finding q_{in} for your Gaussian surface
4. Solve for E.

Infinite line of charge with Gauss' Law

Infinite Line with linear charge density λ



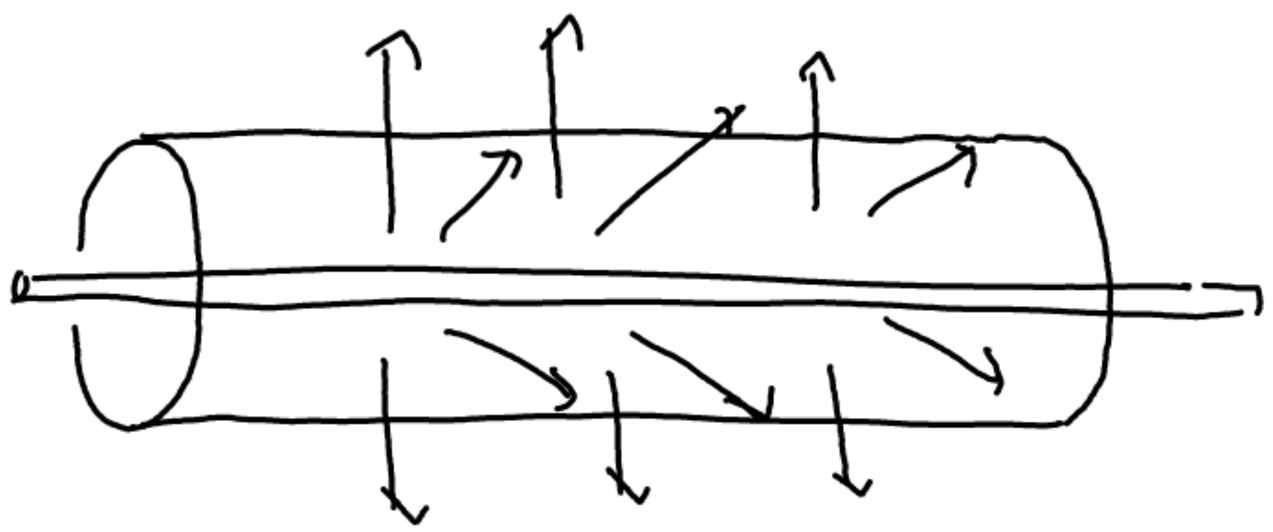
1. identify symmetry

field will be the same everywhere along the line (z-axis)

field points radially outward

cylindrical symmetry

2. Gaussian surface with this symmetry



the E-field is perpendicular to the cylindrical surface

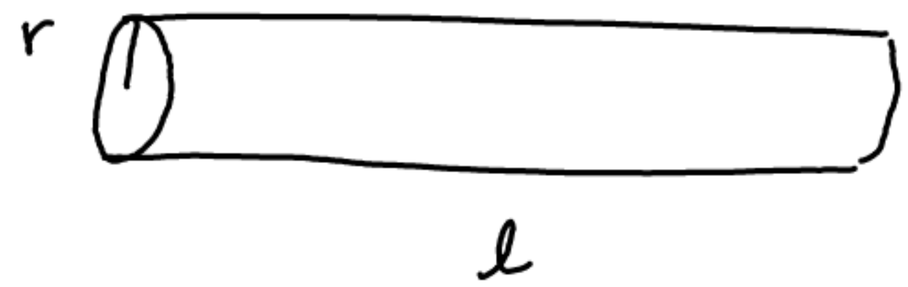
3. Surface area of the cylinder for $\vec{E} \cdot \vec{A}$

note Flux through the ends is zero

so we only need the surface area of

the sides

$$A = 2\pi r \ell$$



Then $\oint \vec{E} \cdot d\vec{A} = E(r) \cdot 2\pi r \ell$

4. Find q_{enc} inside the cylinder

A section of length ℓ is inside the surface

$$q_{enc} = \lambda \cdot \ell$$

5. Use Gauss' Law to find $E(r)$

$$\oint \vec{E} \cdot d\vec{A} = q_{enc}/\varepsilon_0$$

$$E(r) \cdot 2\pi r \cancel{\ell} = \lambda \cancel{\ell}/\varepsilon_0$$

$$\vec{E}(r) = \frac{\lambda}{2\pi r \varepsilon_0} \hat{r}$$

← put the direction back on at the end

We can compare this to what we got by integrating dE:

$$E(r) = \frac{\lambda}{4\pi\varepsilon_0}\,\frac{1}{r}\,\frac{L}{\sqrt{L^2/4 + r^2}}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\,\frac{1}{r}\,\frac{\kappa}{\kappa\sqrt{1/4 + r^2/L^2}}$$

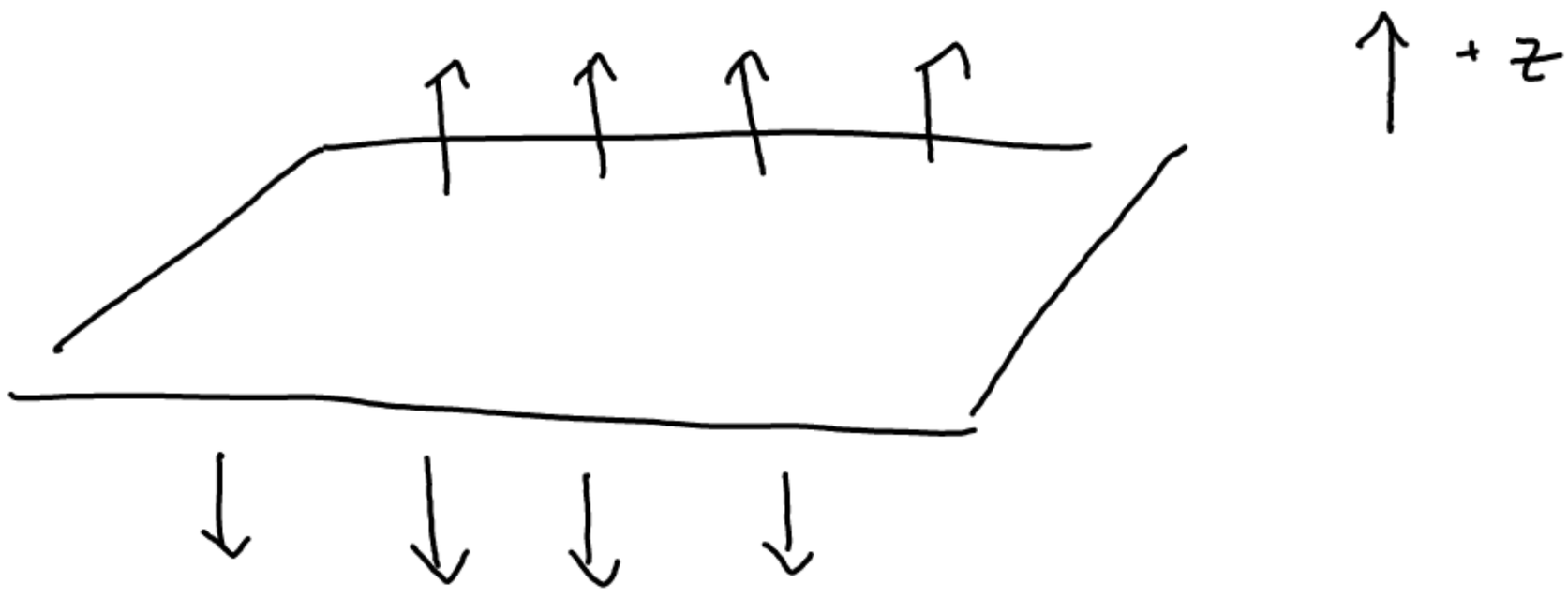
take $L \to \infty$

$$E(r) \sim \frac{\lambda}{4\pi\varepsilon_0}\,\frac{1}{r}\,\frac{1}{\sqrt{1/4 + 0}}$$

$$\boxed{E(r) = \frac{\lambda}{2\pi r \varepsilon_0}}$$ same result!

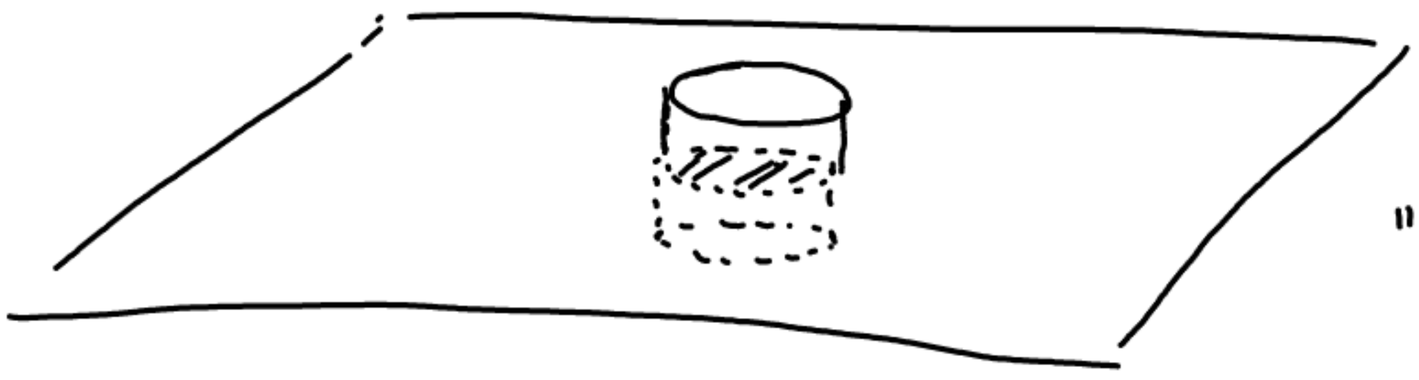
Infinite plane of charge with Gauss' Law

Infinite Sheet



1. Identify symmetry: it's a plane

2. So we want a Gaussian surface with planes parallel to each side of the sheet

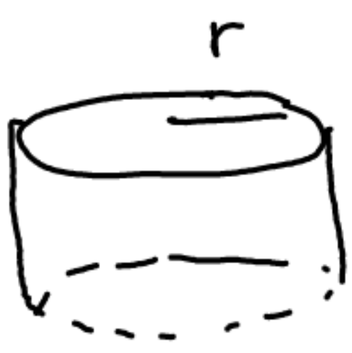


"pill box"

the cylinder's ends are parallel to the plane + its field

the cylinder's sides are ⊥

(no $\vec{E} \cdot \vec{A}$ contribution)



end - area $A = \pi r^2$

3. $\vec{E} \cdot \vec{A}$ from each side is $E(z) \cdot A$

4. $q_{enc} = \sigma \cdot A$

5. Gauss' Law

$$\oint \vec{E} \cdot d\vec{A} = q_{enc} / \varepsilon_0$$

$$2 E(z) \cdot \cancel{A} = \sigma \cancel{A} / \varepsilon_0$$

$$\vec{E}(z) = \frac{\sigma}{2\varepsilon_0} \hat{z}$$

same result we got by integrating!

Notice that twice now the surface area from our Gaussian surface has been canceled by the form of q_{enc}.

This is by design! We get this by choosing the right symmetry.

One more thing: symmetry only works when our charge distribution is "ideal"

↖ infinite for lines / planes

spherical or point-like for spheres

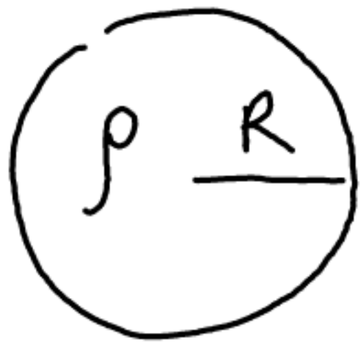
We can make real objects closer to ideal by getting really close or making them really big.

(or really far away, really small)

Solid sphere of charge with Gauss' Law

- Outside the sphere
- Inside the sphere

Sphere of charge



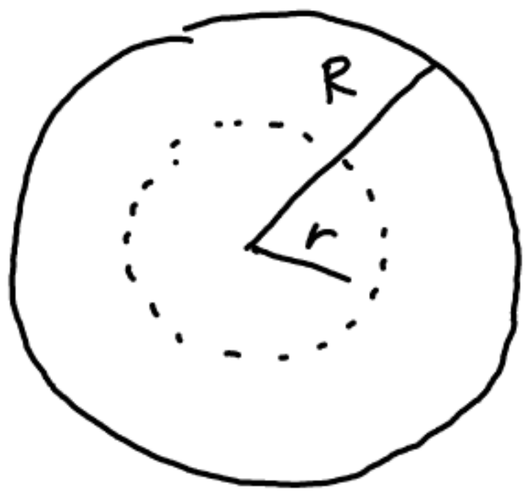
uniform charge density ρ

Find $E(r)$ for <u>all</u> r

1. spherical symmetry

2.

$r < R$



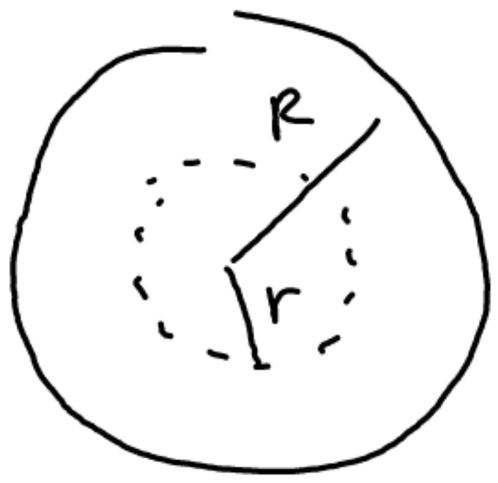
Gaussian surface inside sphere

$r = R$ Gaussian surface at charged sphere surface

$r > R$



$r < R$



$$Q \rightarrow \rho = \frac{Q}{V}$$

$$= \frac{Q}{4/3\,\pi R^3}$$

$$EA = q_{enc}/\varepsilon_0$$

A Gaussian surface $= 4\pi r^2$

$$q_{enc} = \rho V_{enc} = \rho \cdot \frac{4}{3}\pi r^3$$