Defining Gauss’ Law

PHY 114-01

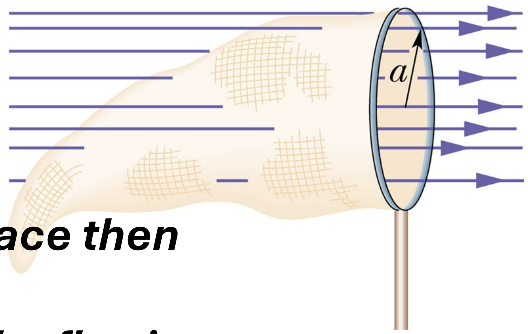
Fall 2026

Agenda

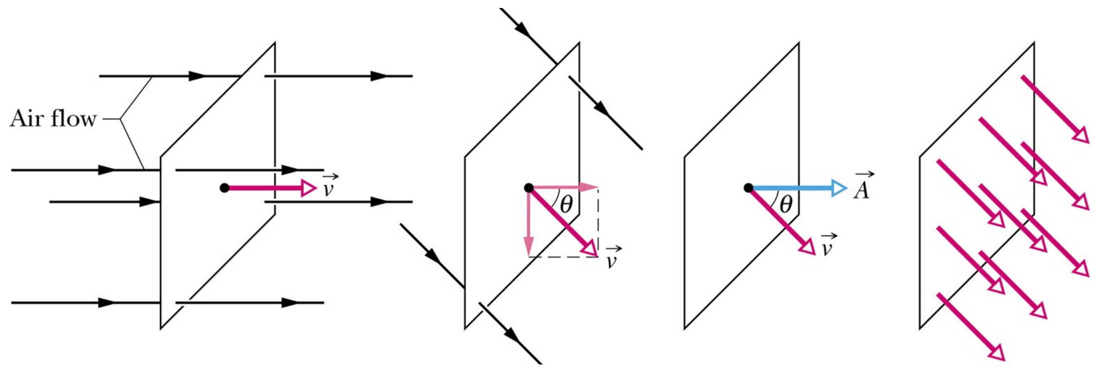
- Last time:
  - Electric Flux
- This time:
  - Calculating flux
  - Gauss' Law
  - Field from a point charge (again)

Concept of Flux

Consider wind flow through the net . Amount of air depends on **area**, and the **angle** between the area and the flow

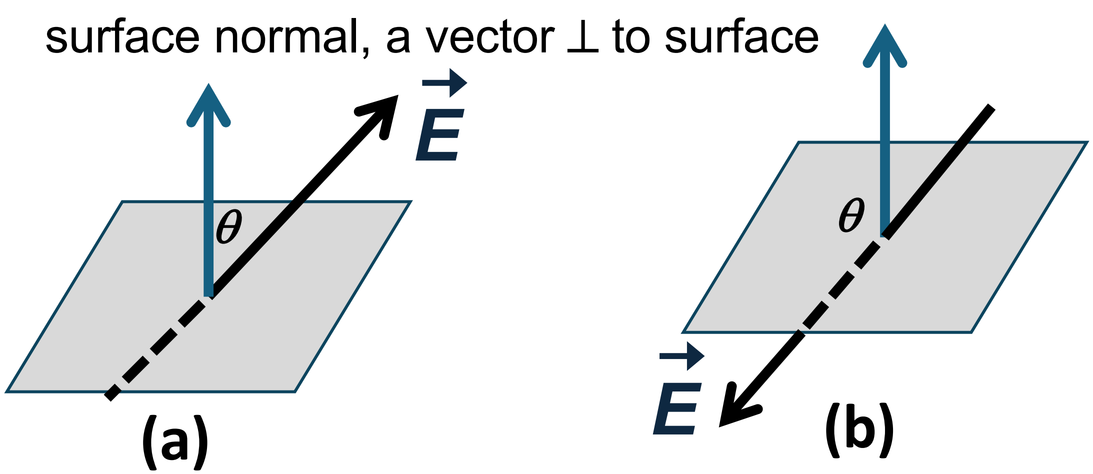


- ***if wind is perpendicular to the surface then the flow, or flux, is a maximum***
- ***if it is parallel to the surface then the flux is zero***
- ***for other angles, we need the component of the wind that is perpendicular to the surface***



Electric Flux

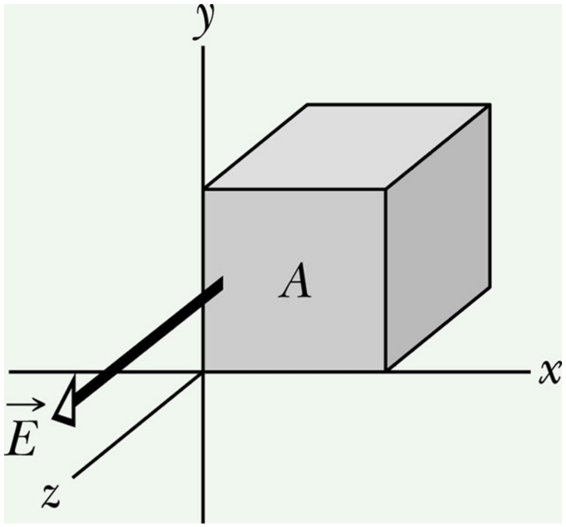
- **definition of electric flux ϕ**
  - $\phi = EA\cos\theta$;   **units: N m^2/C**
- **electric flux is a scalar, not a vector**
  - **it doesn't have direction, but does have a sign**
    - **positive, going *outwards,* (a)**
    - **negative, going *inwards,* (b)**



Checkpoint: Electric flux

The figure shows a cube in a uniform electric field along *z*-axis.

What is the electric flux through the front surface (in *xy* plane)?

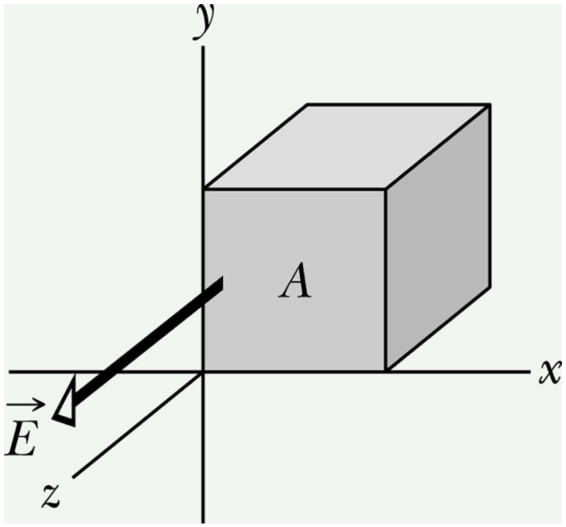


(a) *EA*
(b) -*EA*
(c) 0
(d) 0
(e) None of the above

Checkpoint: Electric flux

The figure shows a cube in a uniform electric field along *z*-axis.

What is the electric flux through the top surface (in *zx* plane)?



(a) *EA*
(b) -*EA*
(c) 0
(d) 0
(e) None of the above

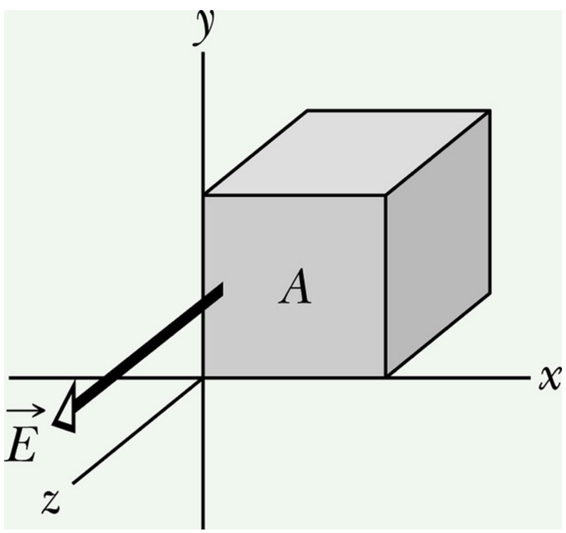
Flux can get us the electric field magnitude

- If we can find the flux through a surface, and we know the surface area, we can back out the magnitude of the field
- IF: the field is uniform magnitude across the surface
- IF: the field is normal to the surface or has some fixed angle relationship

Checkpoint: Electric flux

The figure shows a cube in a uniform electric field along *z*-axis.

What is the electric flux through the entire cube?



(a) *EA*
(b) -*EA*
(c) 0
(d) 0
(e) None of the above

What is the flux through the entire cube?

- If the field has some source or sink that is not inside the cube, the total flux through the cube will be zero
- This is just continuity – you could think of water flowing through an imaginary sphere. What goes in must go out!

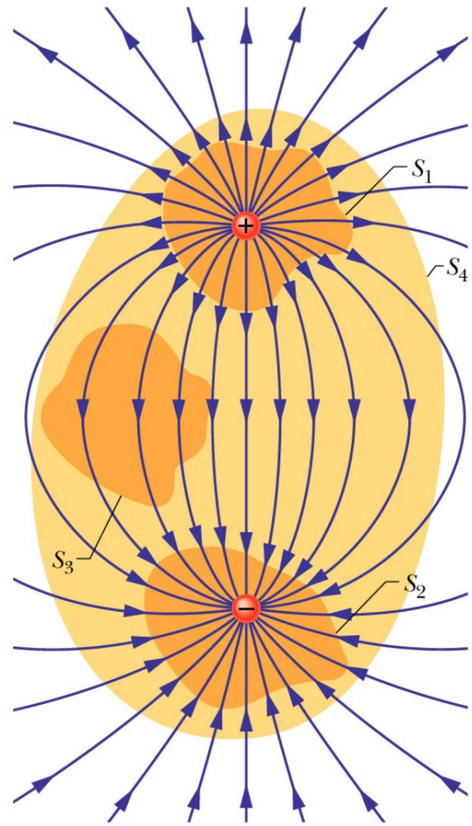
Sphere *enclosing* a point charge

- The net flux is no longer zero!
- Since there is a source, the field doesn't simply pass through

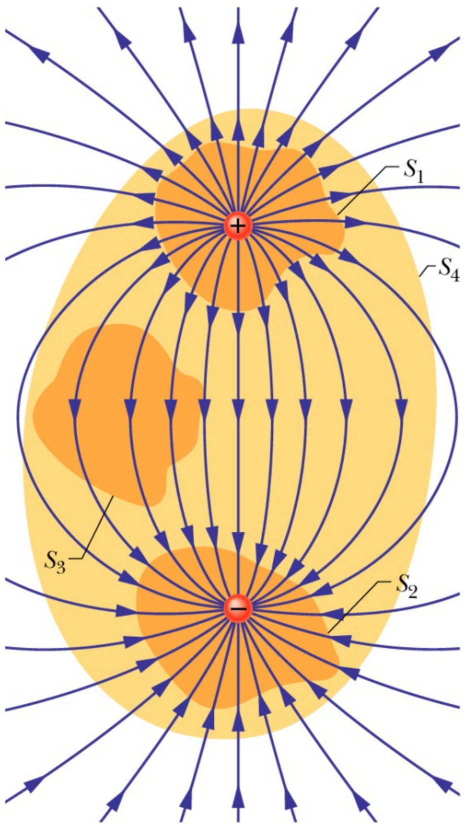
- The flux through the sphere divided by the surface area is the electric field magnitude at the radius of the sphere.

Consider 2 charges (equal in magnitude but opposite sign) and several different surfaces

- **surface around the pos charge**
- **surface around the neg charge**
- **surface around no charge**
- **surface around both charges**

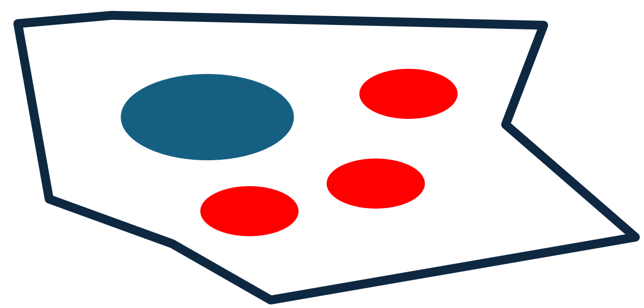


- **surface around the pos charge**
  - **the E field is outwards so the flux is positive**
- **surface around the neg charge**
  - **the E field is inwards so the flux is negative**
- **surface around no charge**
  - **the E lines enter and leave the surface, so no net flux**
- **surface around both charges**
  - **as many E lines leave as enter the surface, no net flux**



ConcepCheck   Enclosed Charge

Several objects are surrounded by a Gaussian surface and the total flux through the surface is zero. What can we say about the enclosed charge ?

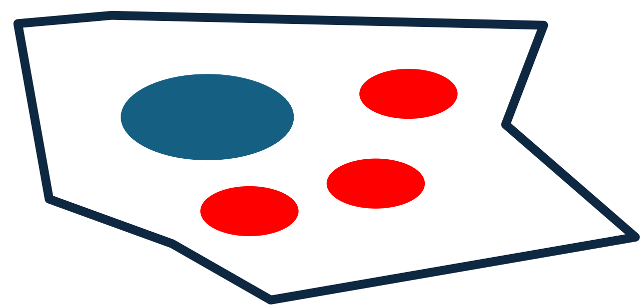


A. **total charge is negative**
B. **total charge is positive**
C. **total charge is zero**
D. **there is not enough information**

The fact that the flux through the surface is zero tells us that the net enclosed charge is zero

ConcepCheck ***Enclosed Charge***

Several objects are surrounded by a Gaussian surface and the total flux through the surface is zero. What can we say about the enclosed charge ?



- **A.** **there is equal positive and negative charge**
- **B.** **there are no charges**
- **C.** **the charges are uniformly distributed inside**
- **D.** **there is not enough info**

The fact that the flux through the surface is zero tells us that the net enclosed charge is zero

Gauss' Law

$$\Phi_E = q_{in}/\epsilon_0$$

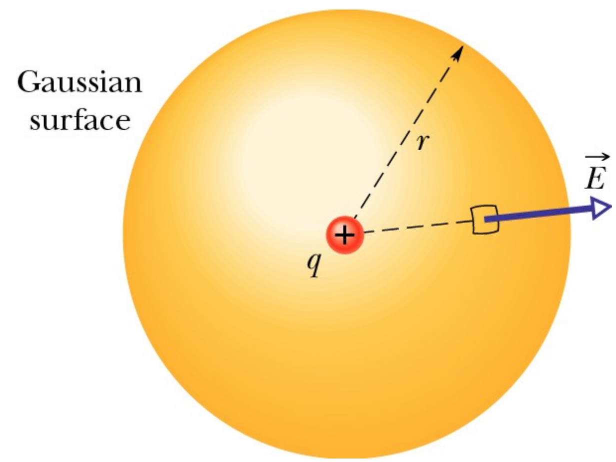
The flux of the electric field through a closed surface

is equal to

The charge enclosed by that surface (divided by ϵ_0)

- We call the closed surface a "Gaussian surface"
- The surface can be anything as long as it's closed
- We will chose surfaces that are
  - 1. matching the symmetry of our charge distribution
  - 2. easy to calculate surface area for

Gauss' Law



$$\Phi_E = \oint \vec{E} \cdot \overrightarrow{dA}$$

- Integral over the closed surface
- For every tiny bit of surface $d\vec{A}$ is the surface area element times the local normal vector.

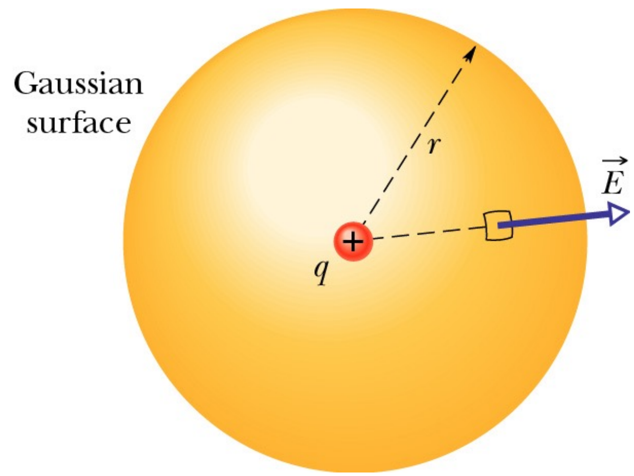
For the sphere around a point charge, E and A are automatically parallel

$$q = \varepsilon_0 \oint EdA$$

$$q = \varepsilon_0 E(4\pi r^2)$$

Gauss' Law & Coulomb's Law

Consider a point charge q surrounded by a spherical Gaussian surface of radius r



- **Gauss' Law can be written as**

$$q = \varepsilon_0 E(4\pi r^2)$$

- **rearranging this gives**

Coulomb's Law

$$E = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q}{r^2}$$

OR

$$E = k\frac{q}{r^2}$$

Procedures for using Gauss' Law

1. Identify the symmetry of your charge distribution.
2. Draw a Gaussian surface with that symmetry around the charges. E should be normal to the surface.
3. Write down the surface area A of your Gaussian surface.
4. Write Gauss' Law $EA = \frac{q_{in}}{\epsilon_0}$. Note that this form only works if you were successful in part 2.
5. Solve for E.

Common symmetries

- Cylindrical
  - Line of charge
  - Cylinder of charge
- Planar
  - Plane of charge
- Spherical
  - Hollow “shell” of charge
  - Solid sphere of charge
  - Point charge

- Many things look like cylinders or planes when you get up close
- Many things look like point charges or spheres when you get far away