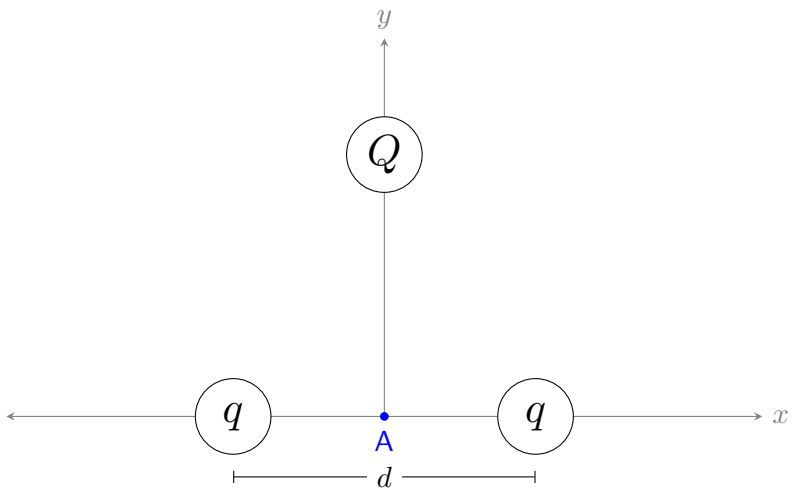
Physics 114 Practice Problems for Exam 2

October 29, 2026

SOLUTIONS

1. Three point charges, two of charge q and one of charge Q, are arranged in a fixed equilateral triangle with side length d, as shown in the diagram below.

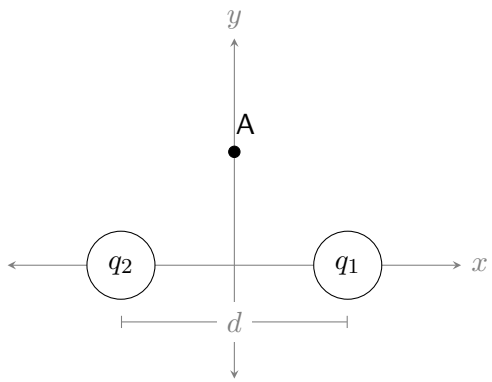


The two charges q are first fixed before the charge Q is brought in from infinity. Compute the work W an external force exerts on the charge Q in order to bring it from infinity to its fixed place in the above triangle.

The work an external force must exert on the charge Q is the difference in electric potential energy of its final position from its initial position. Initially the potential is zero. At the final location, the charge Q is a distance d from both charges q, so the potential there is $V_f = \frac{2k_eq}{d}$. Therefore, this work is

$$W = \Delta U = Q\Delta V = \frac{2k_eQq}{d}.$$

2. Two point charges $q_1 = +Q$ and $q_2 = -2Q$ are separated by a distance d apart, centered about the origin. Point A $(0, a)$ sits on the y-axis midway between them, as in the diagram.



Take the electric potential to be zero at infinity. Find the potential at point A in terms of the variables given and any fundamental constants.

Because potential is a scalar, we can add the potentials at A due to each of the charges:

$$\begin{aligned} V_A &= k\left(\frac{q_1}{r_{A1}} + \frac{q_2}{r_{A2}}\right) \\ &= k\left(\frac{Q}{\sqrt{a^2 + (d/2)^2}} + \frac{-2Q}{\sqrt{a^2 + (d/2)^2}}\right) \\ &= \frac{k}{\sqrt{a^2 + (d/2)^2}}(Q - 2Q) \end{aligned}$$

So we get

$$V_A = -\frac{kQ}{\sqrt{a^2 + (d/2)^2}}$$

3. An electron travels in from infinity, initially traveling to the right at a speed v. It eventually encounters four more stationary electrons arranged in a square with side length d, as shown in the figure below. If the electron happens to stop directly in the center of this square, then what is the side length d?



This problem is most easily solved by conservation of energy, since in both cases it is a simple matter to find the kinetic and potential energies. Since the square of charges is staying the same throughout the problem, we need only consider the kinetic and potential energy of the traveling electron. Its initial kinetic energy is $K_i = \frac{1}{2}m_e v^2$, where m_e is the mass of the electron, and its final kinetic energy is $K_f = 0$ (since it ends up at rest). Its potential energy at any point is $U = q_e V$, where the electric potential is

$$V = k_e q_e \left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4} \right),$$

where r_i is the distance to the i^{th} charge in the square. The electron is initially at infinity, where the potential identically vanishes, so its initial potential energy is $U_i = 0$. At the end, the traveling electron stops in the middle of the square, which is equidistant to the four fixed point charges, namely $r_i = \frac{d}{\sqrt{2}}$. Thus, the final potential energy of the electron is

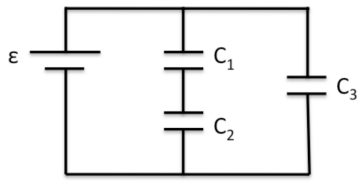
$$U_f = q_e V_f = q_e \left(\frac{4 k_e q_e}{r} \right) = \frac{4\sqrt{2} k_e q_e^2}{d}.$$

Then, conservation of energy becomes

$$K_i + U_i = K_f + U_f \quad \Longrightarrow \quad \frac{1}{2}m_e v^2 + 0 = 0 + \frac{4\sqrt{2} k_e q_e^2}{d},$$

and therefore the side length of the square is

$$d = \frac{8\sqrt{2} k_e q_e^2}{m_e v^2}.$$

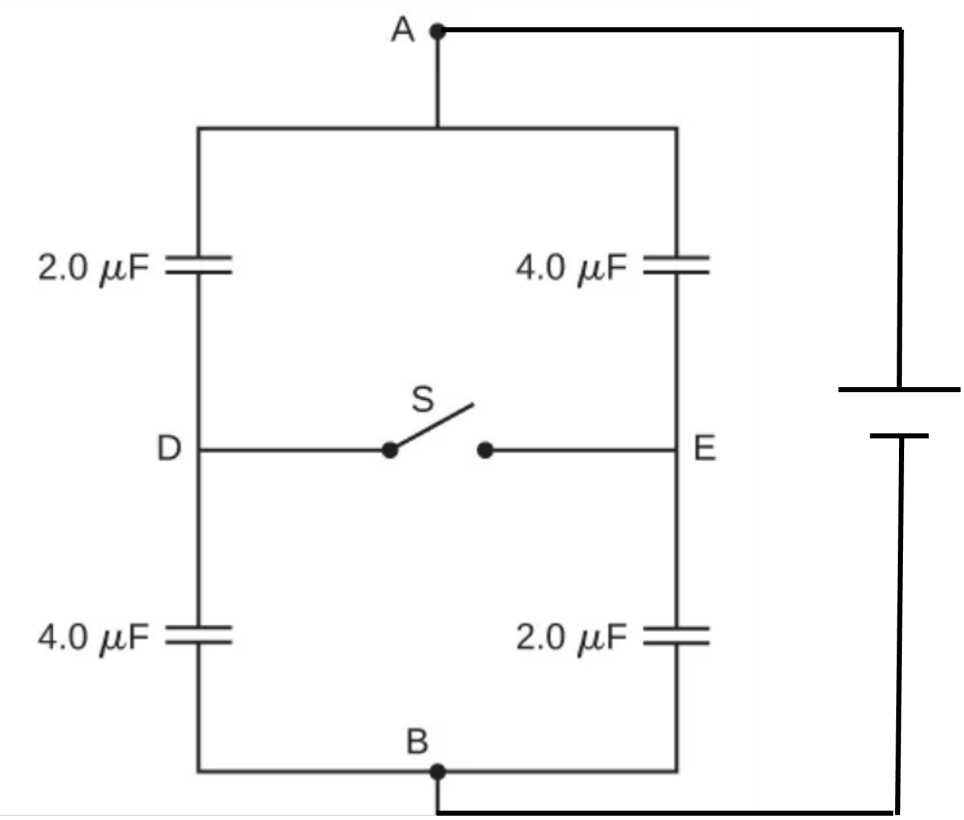


4. Three identical capacitors, each with capacitance C, are connected to a battery as shown, and left for a long time (until equilibrium is reached).

   What is the charge on each individual capacitor C_1, C_2, and C_3?

   We need to calculate the equivalent capacitance to find the total charge. The series combination of C_1 and C_2 are in parallel with C_3. $C_1 = C_2 = C_3 = C$:
   The easiest approach to this is to think about the potential differences across each capacitor. C_3 has the full emf of the battery across it, so $\Delta V_3 = \mathcal{E}$ and $Q_3 = C_3 \Delta V = C\mathcal{E}$.
   C_1 and C_2 are identical capacitors, so they must have the same potential difference across them: $\Delta V_1 = \Delta V_2$. Furthermore, $\Delta V_1 + \Delta V_2 = \mathcal{E}$, so $\Delta V_1 = \Delta V_2 = \frac{\mathcal{E}}{2}$ and $Q_1 = Q_2 = \frac{C\mathcal{E}}{2}$.

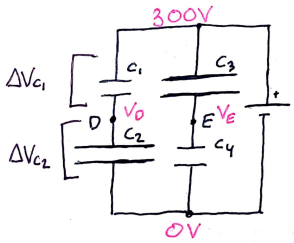
   So each of C_1 and C_2 has a charge $\frac{C\mathcal{E}}{2}$ and C_3 has a charge of $C\mathcal{E}$. Plainly, these don't add up to $\frac{3C\mathcal{E}}{2}$! What's going on? This apparently weird behavior arises because each of the series capacitors carries a charge equal to the total charge of the series combination. Only the upper plate of C_1 and the lower plate of C_2 are connected to the battery, so each of these plates has an equal and opposite charge. The net charge on the combination (lower plate of C_1 and upper plate of C_2) is not connected to the battery, so it must carry a net charge of zero.



5. The network of capacitors shown below are all uncharged when a 300-V voltage source is connected between points A and B as shown, with the switch S open.

   (a) What is the potential difference $V_E - V_D$ after equilibrium is reached?

   We can think of the two branches separately, each as a pair of series capacitors with a total voltage of 300 V across them. Let's take the zero of potential to be the bottom of the circuit (point B).



   To find V_D, we need to figure out ΔV_{C2}. By Kirchhoff's loop rule, $\Delta V_{C1} + \Delta V_{C2} = \mathcal{E} = 300$ V. Because C_1 and C_2 are in series, they have the same charge, which we will call Q. So, we can use our favorite capacitor equation $Q = CV$, along with the loop rule.

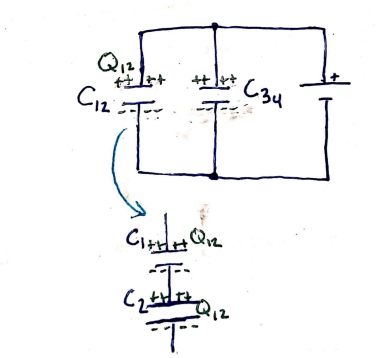
$$Q = C_1 \Delta V_{C1} = C_2 \Delta V_{C2}$$

$$\Delta V_{C2} = \frac{C_1}{C_2} \Delta V_{C1}$$

Conveniently, $\frac{C_1}{C_2} = \frac{1}{2}$ in this case. Therefore:

$$\Delta V_{C2} = \frac{1}{2}\Delta V_{C1}$$

We know that the total $\Delta V_{C1} + \Delta V_{C2} = 300$ V, so $\Delta V_{C2} = 100$ V and $\Delta V_{C1} = 200$ V.
We have now found $V_D = \Delta V_{C2} = 100$ V.
Now we need to find V_E. The other branch is exactly the same, except now the larger capacitor is on the top and the smaller is on the bottom. We can repeat our earlier calculation to find that $\Delta V_{C3} = 100$ V and $\Delta V_{C4} = 200$ V, and $V_E = \Delta V_{C4} = 200$ V.
So, V_E is 100 V higher than V_D, and $V_E - V_D = 100$ V.
You can also do this problem by working out the total charge on the capacitors (creating a C_{eq} for all four capacitors) and then recognizing that half the charge goes to the left branch and half to the right branch, and that *within a single branch, both capacitors have the same charge*. This approach yields the same solution but requires a few more steps.



(b) Now the switch S is closed. What is the potential at point E after equilibrium is reached?

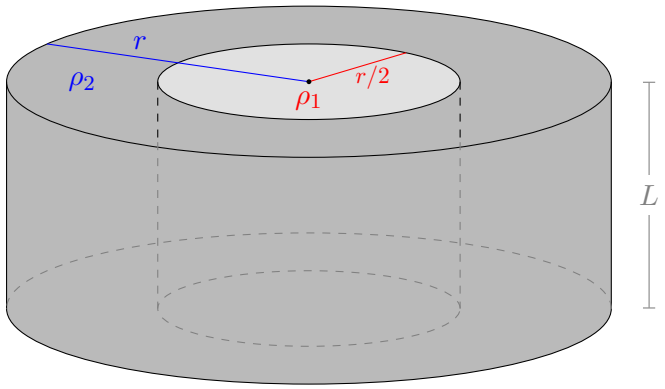
Now we have a different situation: C_1 is in parallel with C_3, and the pair of them are in series with the parallel combination of C_2 and C_4. We can find the equivalent capacitances $C_{13} = C_{24} = (2 + 4)\ \mu\text{F} = 6\ \mu\text{F}$. The two equivalent capacitances have the same charge, which we shall call Q, because they are in series, so we find:

$$Q = C_{13}\Delta V_{13} = C_{24}\Delta V_{24}$$

$$\Delta V_{13} = \Delta V_{24}$$

We know that the total voltage is 300 V, and the voltage at point E is half of the total, and the answer is 150 V.
As before, you could do this part of the problem in terms of charge, but it would require extra steps.

6. A cylindrical resistor of total radius r and length L is constructed out of two different metals. The inner part of the resistor consists of a solid cylinder of radius $r/2$ and length L made of one metal with resistivity ρ_1. Then, this core is surrounded by another cylindrical shell between radius $r/2$ and radius r made of a second metal with resistivity ρ_2; see the diagram below.



Compute the resistance R of this bimetallic cylindrical resistor. Check that the special case $\rho_2 = \rho_1$ reduces to the expected result. [Hint: Think about whether the two metals are connected in series or parallel.]

The fact that both resistors have the same potential at the entire top face and the entire bottom face of the cylinder means that this system behaves as a pure-metal cylinder resistor and a pure-metal cylindrical shell resistor connected in parallel. The resistance of the inner cylinder by itself would be

$$R_1 = \frac{\rho_1 L}{A_1} = \frac{\rho_1 L}{\pi(r/2)^2} = \frac{4\rho_1 L}{\pi r^2}.$$

Likewise, the resistance of the outer cylindrical shell by itself would be

$$R_2 = \frac{\rho_2 L}{A_2} = \frac{\rho_2 L}{\pi[r^2 - (r/2)^2]} = \frac{4\rho_2 L}{3\pi r^2}.$$

Adding these two resistors in parallel gives the total resistance as

$$R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{1}{\frac{\pi r^2}{4\rho_1 L} + \frac{3\pi r^2}{4\rho_2 L}} = \frac{4L}{\pi r^2[\frac{1}{\rho_1} + \frac{3}{\rho_2}]},$$

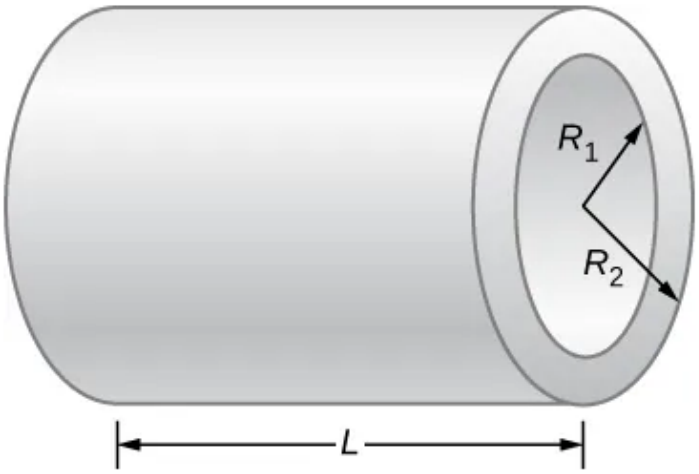
and therefore the resistance of this bimetallic resistor is

$$R = \frac{4\rho_1\rho_2 L}{(3\rho_1 + \rho_2)\pi r^2}.$$

When ρ_2 approaches ρ_1, this result becomes

$$R \xrightarrow{\rho_2 \to \rho_1} \frac{\rho_1 L}{\pi r^2},$$

which is the familiar resistance of a solid cylinder of radius r made from just a single material of resistivity ρ_1.



7. Consider a resistor made from a hollow cylinder of carbon as shown below. The inner radius of the cylinder is $R_1 = 2.0$ mm and the outer radius is $R_2 = 2.1$ mm. The length of the resistor is $L = 10.0$ mm. The resistivity of the carbon is $\rho = 3.5 \times 10^{-5}$ Ωm.

   (a) What is the end-to-end resistance?

   We use $R = \rho \dfrac{L}{A}$. The cross-sectional area of the cylinder is:

   $$A = \pi R_2^2 - \pi R_1^2$$

   We are given the dimensions and the resistivity, so if we plug in numbers in SI units (convert mm to m), we find:

   $$R = \rho \frac{L}{\pi R_2^2 - \pi R_1^2} = .270\Omega$$

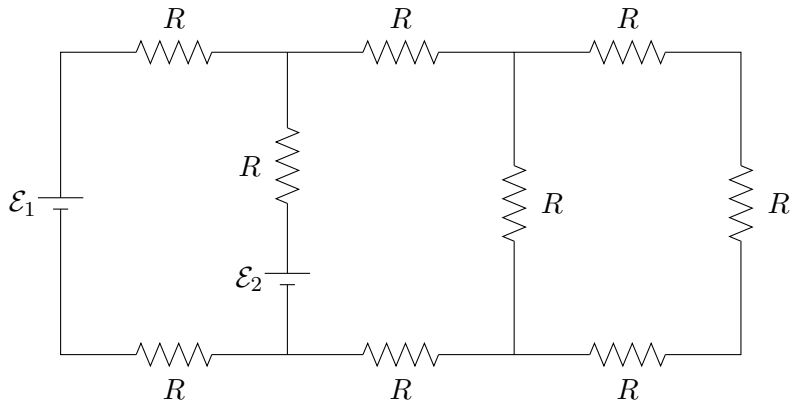
   (b) If 5 Volts were applied across the resistor, how much current would flow through it?

   We use Ohm's Law $V = IR$. The current is $I = \dfrac{V}{R} = 18$ A.

   (c) How much power would be dissipated in the resistor in this case?

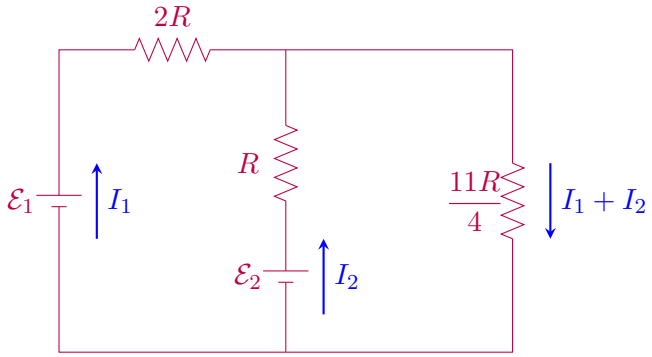
   Power is related to the current and the voltage, $P = IV$. We know both of these quantities from above, so we can calculate that the power dissipated is 90 J/s or 90 W.

8. Consider the following circuit:



Compute the current flowing up through battery $\mathcal{E}_1$. [Hint: Reduce circuit before applying loop rules]

First, combining the resistors that are in series and in parallel results in the equivalent circuit:



Then, applying Kirchhoff's loop rule to the left and the right loops gives

$$\mathcal{E}_1 - 2RI_1 + RI_2 - \mathcal{E}_2 = 0$$
$$\mathcal{E}_2 - RI_2 - \frac{11R}{4}(I_1 + I_2) = 0.$$

The first equation expresses I_2 as

$$I_2 = 2I_1 - \frac{\mathcal{E}_1 - \mathcal{E}_2}{R},$$

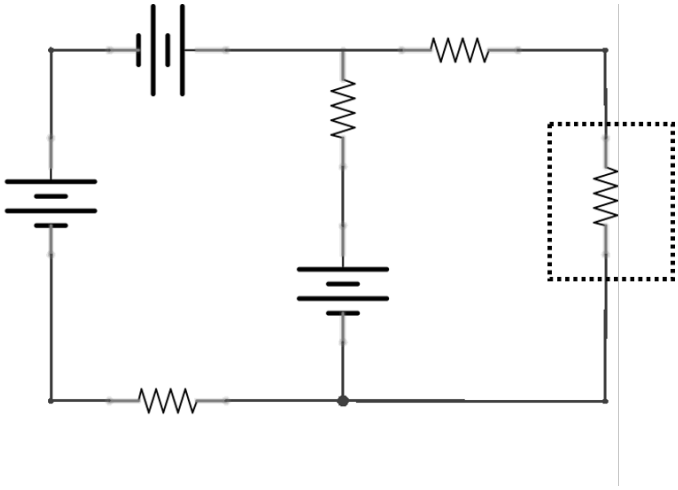
so that the second equation then becomes

$$0 = \mathcal{E}_2 - \frac{11R}{4}I_1 - \frac{15R}{4}I_2 = \mathcal{E}_2 - \frac{41R}{4}I_1 + \frac{15(\mathcal{E}_1 - \mathcal{E}_2)}{4}.$$

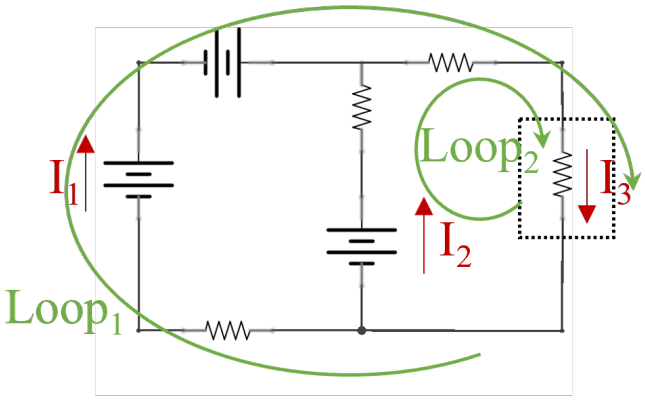
Therefore, the current I_1 flowing up through the resistor $\mathcal{E}_1$ is

$$I_1 = \frac{15\mathcal{E}_1 - 11\mathcal{E}_2}{41R}.$$

9. An electrical circuit is shown below. All resistors have resistance R, and all batteries have voltage V.



What is the current through the resistor indicated by the dashed box? Make clear the direction of current flow.



We set up our loops and currents as shown above, so that all battery voltages will be positive and all resistor voltages will be negative, which is convenient.
We write down the junction law, choosing either the upper or lower 3-way junction (they give the same result):

$$I_3 = I_1 + I_2$$

We write down two loop laws, one for each loop:

$$L_1 : 0 = 2V - 2I_3R - I_1R$$

$$L_2 : 0 = V - I_2R - 2I_3R$$

We want I_3, so let's solve the two loop laws for I_1 and I_2 and then plug those into the junction law.
Working through the algebra, the final answer is

$$I_3 = \frac{3V}{5R} \text{ down.}$$

Because the answer was positive, we know the current flows in the direction we defined, which is downward.