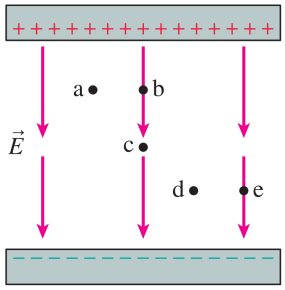
Practice Problems: Electric Potential

Subskills

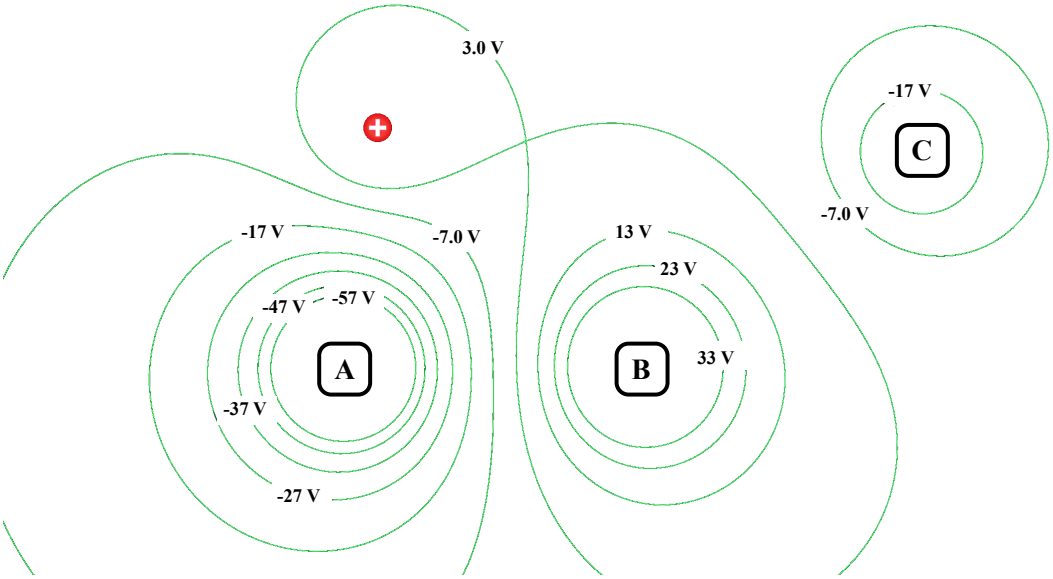
1. Two identical positive charges $+q$ are separated by distance r. The potential energy of this two-charge system is
   (a) $U = kq^2r^2$
   (b) $U = k\dfrac{q}{r}$
   (c) $U = k\dfrac{q^2}{r}$
   (d) $U = k\dfrac{q}{r^2}$
   (e) $U = k\dfrac{q^2}{r^2}$
2. The potential at distance r from a point charge Q is
   (a) $V = k\dfrac{Q}{r}$
   (b) $V = k\dfrac{Q}{r^2}$
   (c) $V = kQr$
   (d) $V = \dfrac{Qr}{k}$
3. Which of the following is true about electric potential (V) and electric potential energy (U)?
   (a) Both depend on the test charge.
   (b) Potential (V) depends only on the source charges; potential energy (U) depends on both source and test charges.
   (c) Potential energy (U) depends only on the source charges; potential (V) depends on both source and test charges.
   (d) Neither depends on the test charge.
4. If the distance between two like charges is doubled, the potential energy of the system
   (a) Doubles.
   (b) Halves.
   (c) Becomes one-quarter of the original.
   (d) Remains the same.

5. Rank in order, from largest to smallest, the potentials V_a to V_e at the points a to e. Note: the $\vec{E}-$field between 2 large parallel plates with equal and opposite surface charge Q is constant.

(a) $V_a = V_b < V_c < V_d = V_e$
(b) $V_a = V_b > V_c > V_d = V_e$
(c) $V_a < V_b = V_c < V_d < V_e$
(d) $V_a > V_b = V_c > V_d > V_e$



6. The figure below shows equipotential lines for an arrangement of four charges. The red charge shown has charge $+q$. The three other charges have charge $-4q$,$+3q$, and $-q$. Match the locations A, B and C with the appropriate charge.



In lecture, we learned that the $E-$field can be calculated from the potential V by taking the gradient: $\vec{E} = -\nabla V$ or $\vec{E}_s = -\frac{dV}{ds}\hat{s}$ in one dimension. Fill in the blanks for the following two questions with the given responses "high" or "low".

(a) high, high

(b) high, low

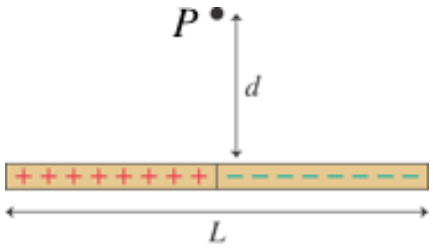
(c) low, high

(d) low, low

7. The gradient points from ___ potential to ___ potential.
8. The electric field points from ___ potential to ___ potential.

9. The left half of the uniformly charged rod in the figure below carries $+Q$, and the right half carries $-Q$. What is the electric potential at point P?

(a) $V < 0$
(b) $V = 0$
(c) $V > 0$



10. Would it require substantially more calculation to calculate the electric field or the electric potential at point P in the previous problem?

(a) Electric field

(b) Electric potential

11. If we brought a point particle with charge $+e$ to point P from infinity, which of the following is true about the change in the particle's electric potential energy ΔU?

(a) $\Delta U < 0$

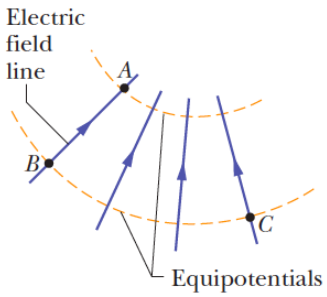
(b) $\Delta U = 0$

(c) $\Delta U > 0$

(d) ΔU depends on the path the particle takes

When an electron moves from A to B along an electric field line the electric field does $W = 1.67 \times 10^{-19}$J amount of work on it. The corresponding change in the electric potential from A to B is $V_B - V_A = 1.00$ V. Find the potential differences shown below; possible answers are 1 V, -1V and 0V.

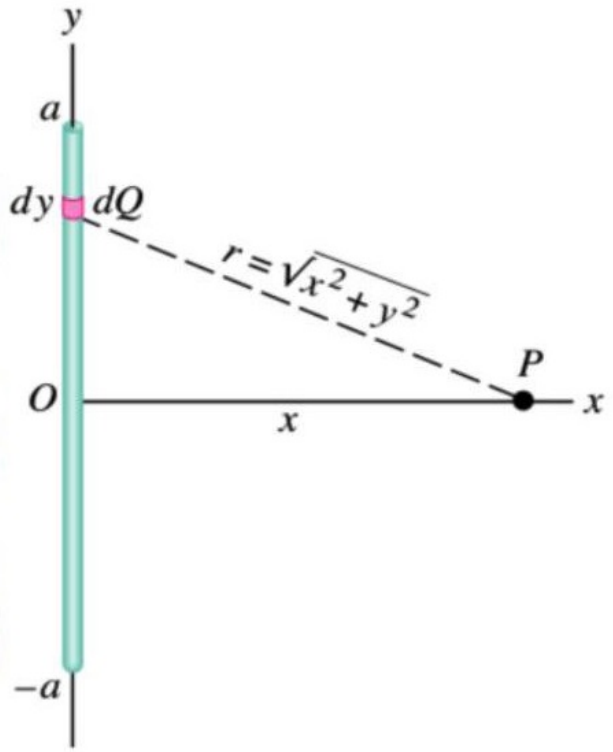
(a) 1 V
(b) 0 V
(c) -1 V



12. $V_B - V_A$
13. $V_C - V_A$
14. $V_A - V_C$
15. $V_C - V_B$
16. $V_B - V_C$

Problems

1. (10 Points) A thin non-conducting rod of length $2a$ lies along the y-axis and has a uniform linear charge density λ, as shown in the figure.



Calculate the electric potential V due to the rod at point P, which lies a perpendicular distance x from the center of the rod. Note that you will need the following integral to solve this problem:

$$\int \frac{dy}{\sqrt{c^2+y^2}} = \ln\left[y + \sqrt{c^2+y^2}\right],$$

where c is a constant.

2. (15 points) When the electric potential depends on more than one variable, we need to use the gradient operator $\vec{\nabla}$ to find the gradient. The gradient operator turns a scalar function $V(x, y, z)$ into a vector:

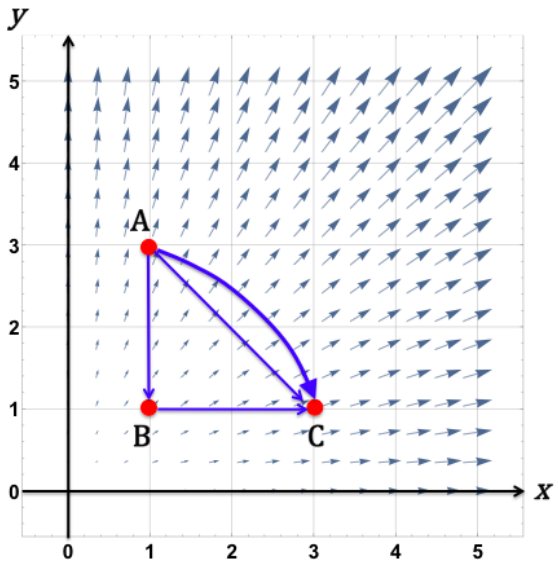
$$\vec{\nabla}V(x, y, z) = \left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z}\right) \tag{1}$$

For example, the gradient of $V(x, y, z) = x^2 + y^2 + z^2$ is $\vec{\nabla}V(x, y, z) = \Big(2x, 2y, 2z\Big)$. The gradient points perpendicular to the equipotentials and towards the point of steepest ascent. The electric field is most generally found from the potential through $\vec{E} = -\vec{\nabla}V(x, y, z)$. Find the electric field from the following electric potentials:

(a) $V(x, y, z) = x + y + z$

(b) $V(z) = \frac{\eta}{2\epsilon_0}(\sqrt{z^2 + R^2} - z)$, where η and ϵ_0 are constants.

3. (15 points) Consider the radial electric field $\vec{E} = \beta(x\hat{x} + y\hat{y})$ and the points $A = (1, 3, 0)$, $B = (1, 1, 0)$, and $C = (3, 1, 0)$, as shown in the figure below.

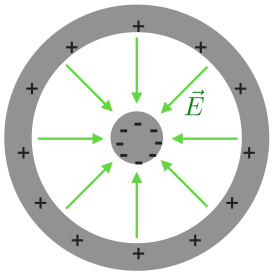


(a) Calculate the potential difference between points A and B.

(b) Calculate the potential difference between points B and C.

(c) Calculate the potential difference between points A and C.

4. The figure below shows a cross section of a device made from concentric **spherical** conductors. The inner sphere has charge $-Q$ and radius R_a. The outer shell has charge $+Q$ and inner radius R_b.



   (a) Use Gauss's law to find the electric field in the region between the conductors, in the region $R_a < r < R_b$.

   (b) Use your result from part (a) to determine the electric potential difference between the conductors.