Practice Problems: Capacitors

Subskills

1. A capacitor is charged with a battery to a potential of V_0. With the battery still connected to the capacitor, the plates of the capacitor are **pushed closer together**. Which of the following is true? Select all that apply.

   (a) The potential difference across the capacitor stays the same.

   (b) The charge on the capacitor stays the same.

   (c) The energy stored in the capacitor increases.

   ★ (a),(c). Since a battery remains connected, the voltage across the capacitor stays the same. Since the distance between the two plates decreases (but the area remains unchanged), the capacitance increases because $C = \epsilon_0 A/d$. Hence, by $C = q/V$, the charge on the capacitor must increase. The energy

   $$U = \frac{1}{2}CV^2.$$

   We have concluded that C increases while V remains unchanged, so U must increase too.

2. A capacitor is charged with a battery to a potential of V_0. After the battery is disconnected from the capacitor, the plates of the capacitor are **pushed closer together**. Which of the following is true? Select all that apply.

   (a) The potential difference across the capacitor stays the same.

   (b) The charge on the capacitor stays the same.

   (c) The energy stored in the capacitor increases.

   ★ (b). Since the battery is disconnected, the charge on the capacitor stays fixed while the voltage across changes. Since the distance between the two plates decreases (but the area remains unchanged), the capacitance increases because $C = \epsilon_0 A/d$. Hence, by $C = q/V$, the voltage must decrease. The energy

   $$U = \frac{1}{2}qV.$$

   We have concluded that V decreases while q remains unchanged, so U must decrease too.

3. A capacitor is constructed of two parallel conducting plates. The plates are separated by a distance d and the space between them is filled with air. There is a battery connected to the plates supplying a potential difference of V_0.

   If the distance between the plates is halved, which of the following statements is correct?

   (a) The potential difference between the plates is doubled.

   (b) The potential difference between the plates is halved.

   (c) The electric charge on the plates is doubled. Correct

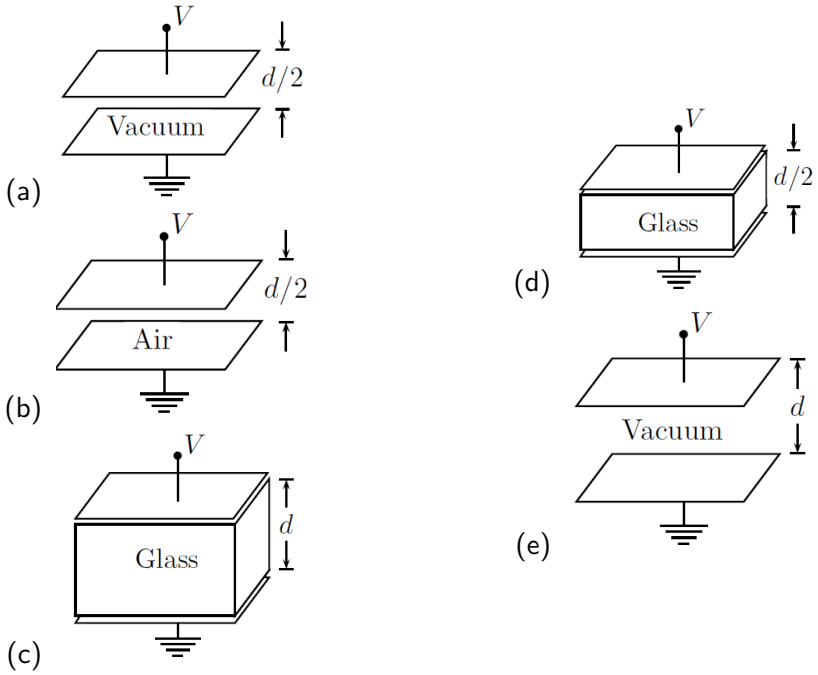
   (d) The capacitance is unchanged.

(e) The electric charge on the plates is halved.

(f) None of the other statements is correct.

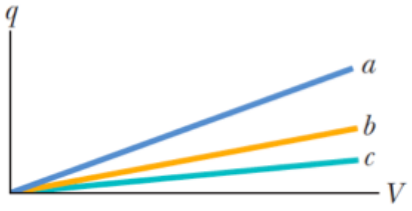
The battery remains connected during the whole process, so the potential difference remains the same throughout. $Q = CV$ and $C = \varepsilon_0 \frac{A}{d}$, so the capacitance is doubled, and the electric charge on the plates is also doubled.

4. The dielectric constant κ is **always**
   (a) greater than 1. ★
   (b) equal to 1.
   (c) less than 1.
   (d) negative
5. When you insert a dielectric into a capacitor with initial electric field E_0, the new field E is
   (a) $E = \kappa E_0$
   (b) $E = E_0/\kappa$ ★
6. Which of the following capacitors, each of which has plates with area A, would store the most charge on the top plate for a given potential V?



The problem actually asks which capacitor has the largest capacitance, since, $Q = CV$. The capacitance for the parallel plate capacitor is $C = \kappa\frac{\epsilon_0 A}{d}$, so the largest κ and smallest distance gives the largest capacitance. Glass, being a dielectric material, has $\kappa > 1$, where the dielectric constant of air is 1. Therefore, the capacitor with the plate distance $d/2$ and glass between plates has highest capacitance.

7. The figure shows a plot of the charge on three different parallel plate capacitors C_1, C_2, C_3 as a function of applied voltage V. The specifications for each capacitor are shown below. Match each line, a, b, and c, with the correct capacitor.



Capacitor	Area	Separation
1	A	d
2	$2A$	d
3	A	$2d$

(a) a $\to$ 1, b $\to$ 2, c $\to$ 3
(b) a $\to$ 2, b $\to$ 3, c $\to$ 1
(c) a $\to$ 3, b $\to$ 1, c $\to$ 2
(d) a $\to$ 1, b $\to$ 3, c $\to$ 2
(e) a $\to$ 3, b $\to$ 2, c $\to$ 1
(f) a $\to$ 2, b $\to$ 1, c $\to$ 3 ★

A parallel plate capacitor has capacitance:

$$C = \frac{q}{V} = \epsilon_0 \frac{A}{d}.$$

So the more the area, the larger the q/V gradient. The more the distance, the smaller the q/V gradient.

8. A parallel plate capacitor has an adjustable plate separation d and an adjustable area A. It is connected to a battery and fully charged to charge Q and voltage V. Which of the following will be true? Select all that apply.

   (a) If we keep the capacitor connected to the battery, adding a dielectric κ increases the Q. Correct.

   (b) If we keep the capacitor connected to the battery, decreasing A will increase Q.

   (c) If we keep the capacitor connected to the battery, increasing d decreases Q. Correct

   (d) Decreasing d will always increase C, regardless of whether or not the capacitor remains connected to the battery. Correct

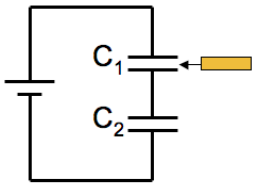
   (e) Increasing A will always increase C, regardless of whether or not the capacitor remains connected to the battery. Correct.

The capacitance can be written as either of the following forms

$$C = \frac{\epsilon_0 A}{d} = QV$$

When we have the battery connected, then the voltage V stays constant. If we disconnect the battery, the total charge Q stays constant.

9. Two identical capacitors are connected in series to a battery as shown in the figure below. If you insert a dielectric into capacitor C_1, the energy stored in capacitor C_2 will:



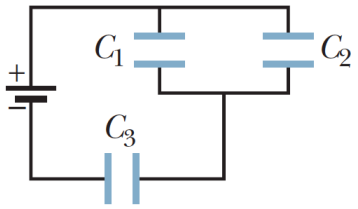
(a) decrease.

(b) remain unchanged.

(c) increase. ★

(d) cannot be determined from the information given.

Consider the circuit with three capacitors as shown in the figure below.



10. Are the capacitors C_1 and C_2 in series or parallel or neither?

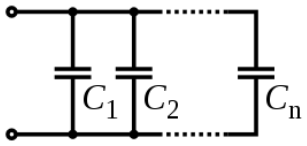
    Parallel. The voltage across them is the same.

11. Are the capacitors C_1 and C_3 in series or parallel or neither?

    Neither. The equivalent capacitor of C_1 and C_2 would be in series with C_3.

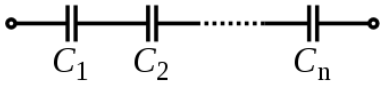
12. Find the equivalent capacitance C_{eq} for N identical capacitors C that are connected in parallel, as shown in the figure. Select all that apply.

    (a) $C_{eq} = NC$ ★
    (b) $C_{eq} = \dfrac{C}{N}$
    (c) $C_{eq} = C_1 + C_2 + C_3 + \cdots + C_N$ ★
    (d) $C_{eq} = \left(\dfrac{1}{C_1} + \dfrac{1}{C_2} + \cdots \dfrac{1}{C_N}\right)^{-1}$
    (e) Cannot be determined from the information given.



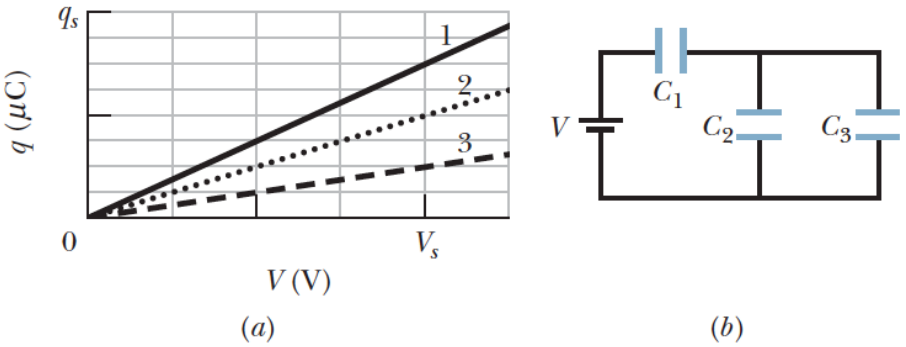
13. Find the equivalent capacitance C_{eq} for N **identical** capacitors C that are connected in series as shown in the figure. Select all that apply.

    (a) $C_{eq} = NC$
    (b) $C_{eq} = \dfrac{C}{N}$ ★
    (c) $C_{eq} = C_1 + C_2 + C_3 + \cdots + C_N$
    (d) $C_{eq} = \left(\dfrac{1}{C_1} + \dfrac{1}{C_2} + \cdots \dfrac{1}{C_N}\right)^{-1}$ ★
    (e) Cannot be determined from the information given.



Problems

1. The figure below shows (a) a plot of the charge that can be stored on each capacitor C_1, C_2, C_3 versus the electric potential V set up across it. The vertical scale is set by $q_s = 16$ μC, and the horizontal scale is set by $V_s = 2$V. It also (b) shows a circuit with these three capacitors and a 6V battery.



(a) (b)

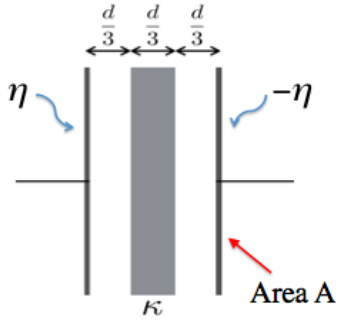
(a) Find the capacitance of C_1, C_2, and C_3.

(b) Find the equivalent capacitance of this circuit. What is the net charge stored on the equivalent capacitor?

(c) Find the charge stored on each capacitor in the circuit.

(d) Find the potential energy U_C stored on each capacitor.

2. Consider a parallel plate capacitor with area A, surface charge density $\pm\eta$, and a total separation distance d. A dielectric material κ is inserted and fills the middle third of the capacitor as shown below.



(a) Find the electric field $\vec{E}$ between the parallel plates with the dielectric inserted.

We separate the problem into the left ($x \in (0, d/3)$), the middle ($x \in (d/3, 2d/3)$), and the right ($x \in (2d/3, d)$) regions.

$$\begin{aligned} \vec{E}_l = \vec{E}_r &= \frac{\eta}{\epsilon_0}\hat{x}, \\ \vec{E}_m &= \frac{\eta}{\kappa\epsilon_0}\hat{x}, \end{aligned}$$

(b) Using $\Delta V = -\int \vec{E} \cdot d\vec{r}$, find the magnitude of the total potential difference $|\Delta V|$ between the two plates.

$$\begin{aligned} \Delta V &= -\int_0^{d/3} \vec{E}_l \cdot d\vec{r} - \int_{d/3}^{2d/3} \vec{E}_m \cdot d\vec{r} - \int_{2d/3}^{d} \vec{E}_r \cdot d\vec{r} \\ &= -\frac{\eta d}{3\epsilon_0} - \frac{\eta d}{3\epsilon_0} - \frac{\eta d}{3\kappa\epsilon_0} \\ &= -\frac{\eta d}{3\epsilon_0}\left(2 + \frac{1}{\kappa}\right). \end{aligned}$$

$$\implies |\Delta V| = \frac{\eta d}{3\epsilon_0}\left(2 + \frac{1}{\kappa}\right).$$

(c) Find the magnitude of charge on each plate in terms of η, A.

By the definition of "surface charge density", the left plate has

$$Q_l = \eta A,$$

and the right,

$$Q_r = -\eta A.$$

(d) Find the capacitance for this capacitor.

Using the answers in parts (b) and (c),

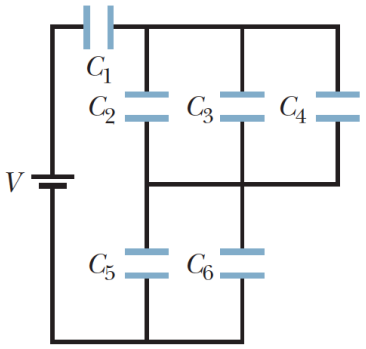
$$C = \frac{Q}{|\Delta V|} = \frac{\eta A}{\frac{\eta d}{3\epsilon_0}\left(2 + \frac{1}{\kappa}\right)} = \frac{3\epsilon_0 A}{d\left(2 + \frac{1}{\kappa}\right)}.$$

(e) How would the capacitance change if the dielectric κ was replaced with a conductor?

A conductor has $\kappa \to \infty$. This is a direct consequence of the zero electric field inside a conductor. So taking the answer in part (d), we see that

$$C \to \frac{3\epsilon_0 A}{2d}.$$

3. Consider a circuit with a battery V that is connected to six identical capacitors C as shown in the figure below.



(a) Find the charge stored on C_4.

See figure below for a redraw of the circuit. Note that C_2, C_3, C_4 are in parallel and can be replaced with equivalent capacitor $C_{234} = C_2 + C_3 + C_4$, and C_5, C_6 are in parallel and can be replaced with equivalent capacitor $C_{56} = C_5 + C_6$. Let the equivalent capacitor for all capacitors be C_{eq}, then the charge across $C_1, C_{234}, C_{56}, C_{eq}$ are all the same.

$$Q_{net} = C_{eq}V = \left(\frac{1}{C_1} + \frac{1}{C_{234}} + \frac{1}{C_{56}}\right)^{-1} V = \left(\frac{1}{C} + \frac{1}{3C} + \frac{1}{2C}\right)^{-1} V = \frac{6}{11}CV = Q_{234}.$$

Since C_2, C_3, C_4 are identical and have the same voltage across them, they share the charge equally, i.e.,

$$Q_4 = \frac{1}{3}Q_{234} = \frac{2}{11}CV.$$

(b) Find the potential difference across C_6.

The potential difference across C_5, C_6 is the same. We have discussed in (a) that $Q_{net} = Q_{56}$. Hence,

$$V_6 = V_{56} = \frac{Q_{56}}{C_{56}} = \frac{\frac{6}{11}CV}{2C} = \frac{3}{11}V.$$

(c) Find the total stored energy in the circuit.

Since we know Q_{net}, C_{eq}, V, we can use any of the three formulas.

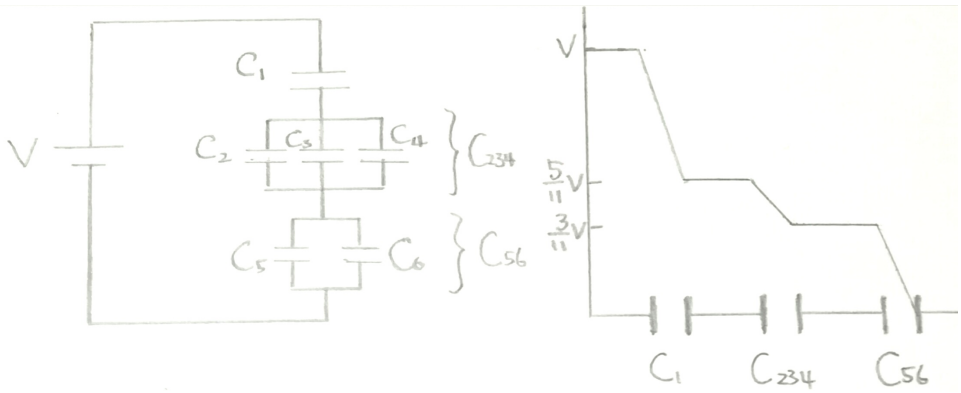
$$U = \frac{1}{2}CV^2 = \frac{3}{11}CV^2.$$

(d) Make a sketch of how the potential varies around the circuit, starting with the positive terminal of the battery at V and ending at the negative terminal at 0.

Note that

$$\begin{aligned} V_1 &= \frac{Q_1}{C_1} = \frac{6}{11}V, \\ V_{234} &= \frac{Q_{234}}{C_{234}} = \frac{2}{11}V, \\ V_{56} &= \frac{3}{11}V, \end{aligned}$$

so the potential falls from V to $5V/11$ to $3V/11$ to 0.



(e) Now a dielectric κ is inserted into capacitor 3, C_3. How does the total stored energy change?

$C_3 \to \kappa C_3$, but the structure of other capacitors remain unchanged. We have

$$C_{eq,f} = \left(\frac{1}{C_1} + \frac{1}{C_{234,f}} + \frac{1}{C_{56}}\right)^{-1} = \left(\frac{1}{C} + \frac{1}{2C + \kappa C} + \frac{1}{2C}\right)^{-1} = \frac{2(2+\kappa)}{8+3\kappa}C.$$

Since the battery is connected, V is also unchanged. Hence,

$$U_{net,f} = \frac{1}{2}C_{eq,f}V^2 = \frac{2+\kappa}{8+3\kappa}CV^2 > U_{net,i}.$$